ALGEBRA II
FIRST SEMESTER REVIEW

Name: Answer Key   Date: ________

A. Solve using **linear combination**.

1. (5) $2x - 3y = 6$
   (2) $5x - 4y = 1$

$10x - 15y = 30$
$-\ 10x - 8y = 2$
$-7y = 28$
$y = -4$

$(-3, -4)$

$5x - 4(-4) = 1$
$5x + 16 = 1$
$5x = -15$
$x = -3$

B. Solve using **Cramer's Rule**.

1. $2x - 3y - z = 1$
   $x - y - 2z = 5$
   $5x - 3y - 4z = -1$

$$|D| = \begin{vmatrix} 2 & -3 & -1 \\ 1 & -1 & -2 \\ 5 & -3 & -4 \end{vmatrix} \begin{matrix} 2 & -3 \\ 1 & -1 \\ 5 & -3 \end{matrix}$$

$(8 + 30 + 3) - (12 + 12 + 5)$
$41 - 29 = \boxed{12}$

$$|D_x| = \begin{vmatrix} 1 & -3 & -1 \\ 5 & -1 & -2 \\ -1 & -3 & -4 \end{vmatrix} \begin{matrix} 1 & -3 \\ 5 & -1 \\ -1 & -3 \end{matrix}$$

$(4 + {}^{-}6 + 15) - (60 + 6 - 1)$
$13 - 65$
-52

$x = \frac{-52}{12} = \frac{-13}{3}$

$$|D_y| = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 5 & -2 \\ 5 & -1 & -4 \end{vmatrix} \begin{matrix} 2 & 1 \\ 1 & 5 \\ 5 & -1 \end{matrix}$$

$(-40 + {}^{-}10 + 1) - (-4 + 4 - 25)$
$-49 - (-25)$

$\frac{-24}{12} = -2$

$$D_z = \begin{vmatrix} 2 & -3 & 1 \\ 1 & -1 & 5 \\ 5 & -3 & -1 \end{vmatrix} \begin{matrix} 2 & -3 \\ 1 & -1 \\ 5 & -3 \end{matrix}$$

$(2 - 75 - 3) - (3 - 30 - 5)$
$-76 - (-32)$

$z = \frac{-44}{12} = \frac{-11}{3}$

$\frac{-11}{3}$

C. Simplify completely.

1. $\sqrt{200}$

$\sqrt{100}\sqrt{2}$

$10\sqrt{2}$

2. $\sqrt{-20}$

$2i\sqrt{5}$

3. $\sqrt{\frac{5}{18}}$

$\frac{\sqrt{5}}{3\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{10}}{6}$

4. $2\sqrt{3} \bullet 3\sqrt{-6} \bullet \sqrt{-2}$

$6\sqrt{3}\ i\sqrt{6}\ i\sqrt{2}$

$6i^2\sqrt{36}$

$36i^2 = -36$

5. $(-3i)^2$

$9i^2$

-9

6. $(2-6i)+(-5+3i)+i$

$2-6i+-5+3i+i$

$-3-2i$

7. $(2-6i)(2+3i)$

$4-12i+6i-18i^2$

$4-6i-18(-1)$

$22-6i$

8. $(4+3i)^2$

$(4+3i)(4+3i)$

$16+12i+12i+9i^2$

$16+24i+9(-1)$

$7+24i$

9. $\frac{2}{i}\left(\frac{i}{i}\right) = \frac{2i}{i^2}$

$= \frac{2i}{-1} = -2i$

10. $\frac{2-i}{1+i}$

Omit

11. i^9

$i^2 i^2 i^2 i^2 i$

$(-1)(-1)(-1)(-1)i$

i

D. Solve each of the following by factoring.

1. $x^2 - 13x - 30 = 0$

$(x-15)(x+2) = 0$

$x-15=0 \qquad x+2=0$

$x = 15 \qquad x = -2$

$\{-2, 15\}$

2. $x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \qquad x - 3 = 0$

$x = 3$

$\{0, 3\}$

3. $2x^2 + 9 = 3x^2 - 7$

$9 = x^2 - 7$

$16 = x^2$

$\boxed{\pm 4 = x}$

4. $x^2 - \frac{1}{6}x = \frac{1}{3}$

$6x^2 - x = 2$

$6x^2 - x - 2 = 0$

$(2x+1)(3x-2) = 0$

$\{-1/2, 2/3\}$

5. $x^3 - 27 = 0$

$(x-3)(x^2+3x+9) = 0$

$x - 3 = 0$

$\boxed{x = 3}$

$x^2 + 3x + 9 = 0$

$x = \frac{-3 \pm \sqrt{9 - 4(1)(9)}}{2(1)}$

$x = \frac{-3 \pm \sqrt{-27}}{2}$

$\boxed{x = \frac{-3 \pm 3i\sqrt{3}}{2}}$

6. $3x^3 - 4x^2 + 12x - 16 = 0$

$x^2(3x-4) + 4(3x-4) = 0$

$(3x-4)(x^2+4) = 0$

$3x - 4 = 0$

$\boxed{x = \frac{4}{3}}$

$x^2 + 4 = 0$

$x^2 = -4$

$\boxed{x = \pm 2i}$

E. Solve by **completing the square**.

Q.F.

1. $x^2 - 6x + 4 = 0$

$x^2 - 6x + 9 = -4 + 9$

$(x-3)^2 = 5$

$x - 3 = \pm\sqrt{5}$

$\boxed{x = 3 \pm \sqrt{5}}$

2. $2x^2 - 10x + 1 = 0$

F. Solve using the **quadratic formula**.

1. $2x^2 - 5x - 1 = 0$

$x = \frac{5 \pm \sqrt{25 - 4(2)(-1)}}{2(2)}$

$x = \frac{5 \pm \sqrt{25 + 8}}{4}$

$x = \frac{5 \pm \sqrt{33}}{4}$

2. $3(x+1)^2 = 4(x-2)$

$3(x^2 + 2x + 1) = 4x - 8$

$3x^2 + 6x + 3 = 4x - 8$

$3x^2 + 2x + 11 = 0$

$x = \frac{-2 \pm \sqrt{4 - 4(3)(11)}}{2(3)}$

$x = \frac{-2 \pm \sqrt{4 - 132}}{6}$

$x = \frac{-2 \pm \sqrt{-128}}{6} = \frac{-2 \pm 8i\sqrt{2}}{6}$

$= \frac{-1 \pm 4i\sqrt{2}}{3}$

G. Find the value of the discriminant. State the number and nature of the roots.

1. $3x^2 - 2x + 5 = 0$

$b^2 - 4ac$

$4 - 4(3)(5)$

$4 - 60$

-56 2 Imag. solutions

2. $x^2 - 6x + 9 = 0$

$b^2 - 4ac$

$36 - 4(1)(9)$

$36 - 36$

0

one Real solution

H. For the quadratic equation $x^2 - 10x + c = 0$, find c so that the equation has

1. one real (rational) root.

$b^2 - 4ac = 0$

$100 - 4(1)c = 0$

$100 - 4c = 0$

$100 = 4c$

$25 = c$

2. two rational roots.

$100 - 4(1)c) > 0$

$100 - 4c > 0$

$c = 10$

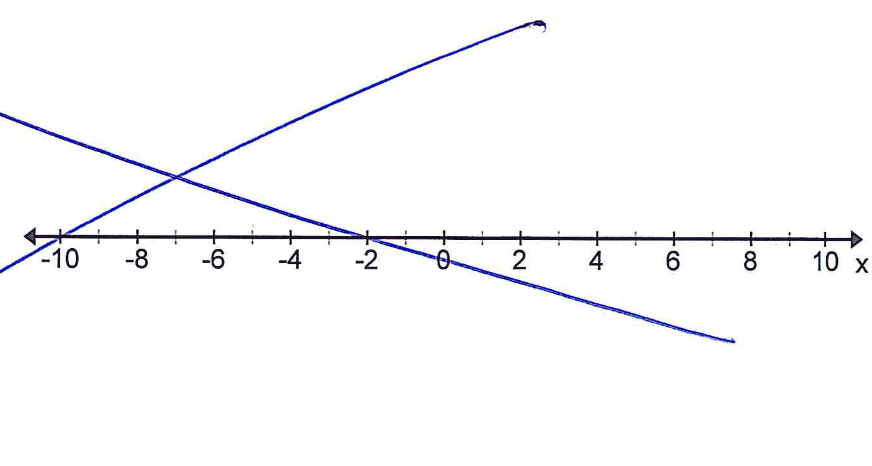
3. two imaginary roots.

$100 - 4c < 0$

$c = 30$

J. Solve.

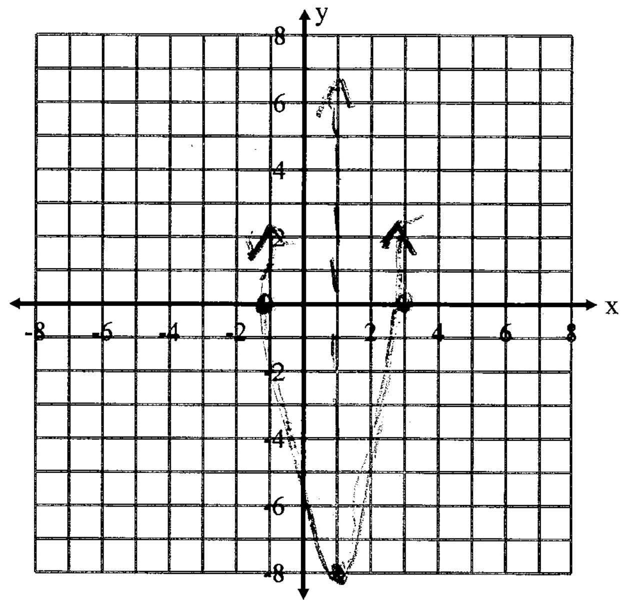
1. $2x^2 + 5x - 12 > 0$



3. $y = 2(x-3)(x+1)$

$y = 2(-2)(2)$

$y = -8$



J. Simplify.

1. $3x^4 \cdot 2x^{-6}$

$6x^{-2} = \frac{6}{x^2}$

2. $(-2x^{-3})^4 \bullet (4x)^{-3}$

$16x^{-12}\left(\frac{1}{64x^3}\right)$

$\boxed{\frac{1}{4x^{15}}}$

3. $\left(\frac{4x^{-3}}{3y^{-5}}\right)^{-2}$

$\frac{4^{-2}x^6}{3^{-2}y^{10}} = \frac{9x^6}{16y^{10}}$

4. $\frac{10x^{-5}y}{12x^{-2}y^{-4}}$

$\frac{5x^2yy^4}{6x^5} = \frac{5y^5}{6x^3}$

K. Divide.

1. $(x^3 + 3x^2 - 13x + 6) \div (x - 2)$

2	1	3	−13	6
		2	10	−6
	1	5	−3	

$\boxed{x^2 + 5x - 3}$

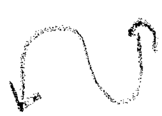
2. $(6x^3 + x^2 + 7x + 10) \div (3x + 2)$

$-\frac{2}{3}$	6	1	7	10
		−4	2	−6
	6	−3	9	4

$\boxed{6x^2 - 3x + 9 + \frac{4}{3x+2}}$

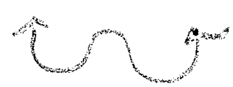
L. Determine the left and right end behavior of the graphs of each given polynomial function.

1. $y = 2x^3 - 8x + 5$



left down
right up

2. $y = x^4 + x^2 - x$



Both up

M. List all of the possible rational zeros of

1. $2x^3 + 7x^2 + 6x - 5 = 0$

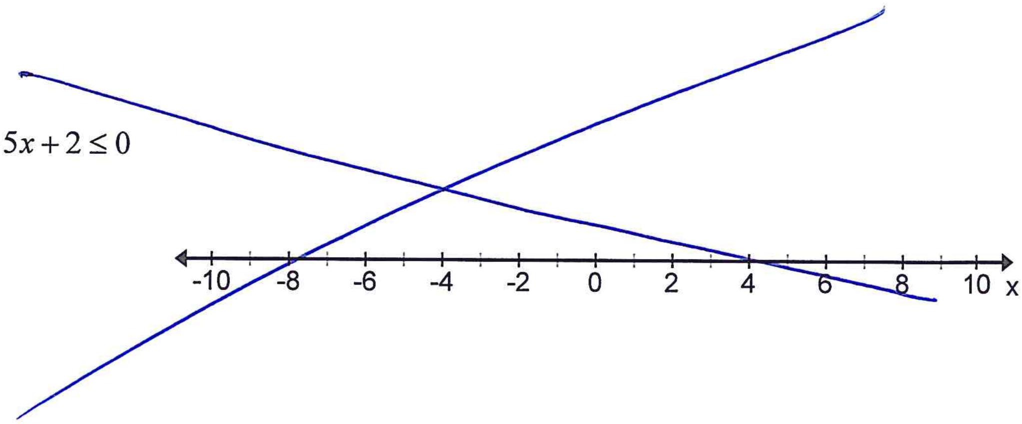
$\frac{\pm 1, \pm 5}{\pm 1, \pm 2}$ $\left\{\pm 1, \pm \frac{1}{2}, \pm 5, \pm \frac{5}{2}\right\}$

2. $6x^5 - 2x^4 + x - 10 = 0$

$\frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1, \pm 2, \pm 3, \pm 6}$

$\left\{\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3}, \pm 5, \pm \frac{5}{2}, \pm \frac{5}{3}, \pm \frac{5}{6}, \pm 10, \pm \frac{10}{3}\right\}$

2. $3x^2 + 5x + 2 \leq 0$



I. Graph.

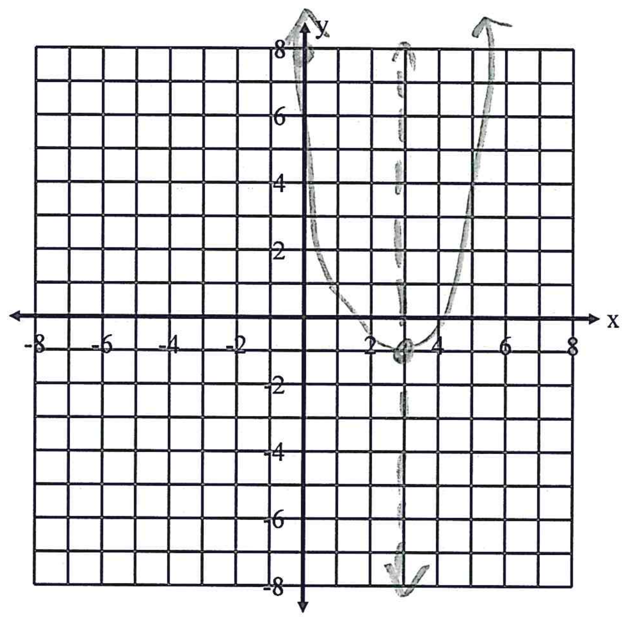
1. $y = x^2 - 6x + 8$

$V\left(\frac{6}{2(1)},\ \right)$

$(3, -1)$

$y = 9 - 18 + 8$

$y = -9 + 8$



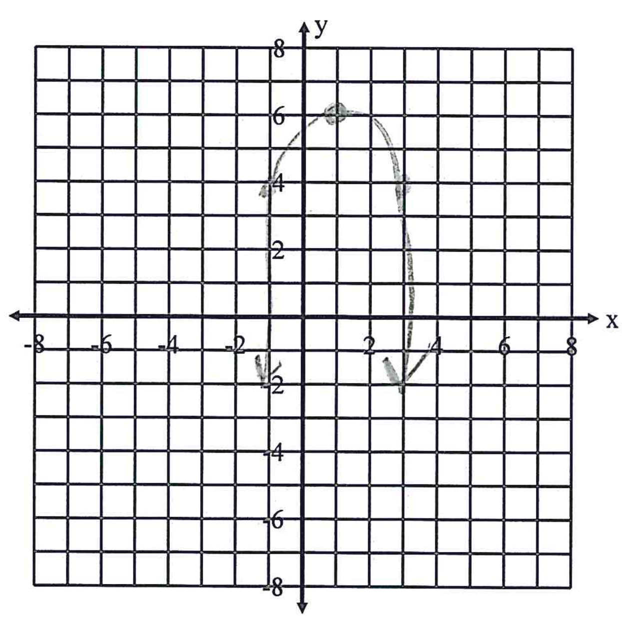
2. $y = -\frac{1}{2}(x-1)^2 + 6$

$y = -\frac{1}{2}(3-1)^2 + 6$

$y = -\frac{1}{2}(4) + 6$

$y = -2 + 6$

$y = 4$



N. Find all the solutions (roots) of

1. $x^3 + 3x^2 - 3x - 9 = 0$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -3 & -9 \\ & & -3 & 0 & 9 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$x^2 - 3 = 0$

$x^2 = 3$

$\boxed{x = \pm\sqrt{3}}$

2. $2x^4 - 3x^3 - 3x - 2 = 0$

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & 0 & -3 & -2 \\ & & 4 & 2 & 4 & 2 \\ \hline -\frac{1}{2} & 2 & 1 & 2 & 1 & 0 \\ & & -1 & 0 & -1 & \\ \hline & 2 & 0 & 2 & 0 & \end{array}$$

$2x^2 + 2 = 0$ $\quad x^2 = -1$

$2(x^2 + 1) = 0$ $\quad x = \pm i$

$\{-\frac{1}{2}, 2, \pm i\}$

M. Write a polynomial equation in standard form which has the given roots.

1. 2, -3, 1

$f(x) = (x-2)(x+3)(x-1)$

$f(x) = (x-2)(x^2+2x-3)$

$x^3 + 2x^2 - 3x - 2x^2 - 4x + 6$

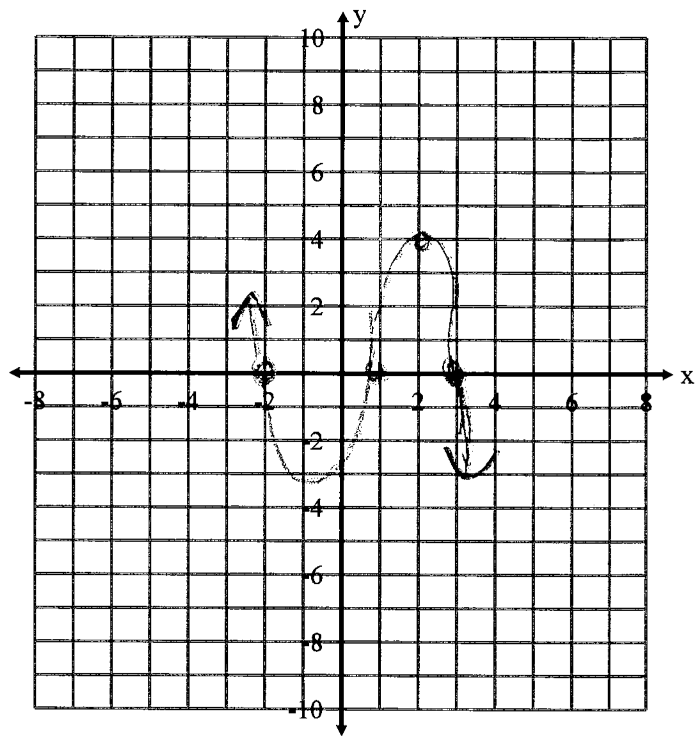
$\boxed{x^3 - 7x + 6}$

2. 1, -1, 2i, -2i

omit

O. Sketch the graph of the function.

1. $y = -1(x-3)(x-1)(x+2)$



$y = -1(-1)(1)(4)$

$y = 4$