Key

Name:

ALGEBRA II - FINAL EXAM REVIEW

This review is to help you get ready for the Final Exam. It will be turned in the day of your final for a grade based on completion and correctness. You must show work on all problems that require some work. For example, if you are asked to multiply and simplify a radical, you should show all of your steps; if you are asked to classify a polynomial, you do not need to show work.

7.1 and 7.2

1.) Simplify

a) $(25)^{\frac{3}{2}} = (\sqrt{25})^3$ $5^3 = 125$

b) $8^{\frac{2}{3}}$ $(\sqrt[3]{8})^2 = 2^2 = 4$

c) $(49)^{\frac{-1}{2}}$ $\frac{1}{49^{1/2}} = \frac{1}{7}$

d) $4^{-\frac{3}{2}}$ $\frac{1}{4^{3/2}} = \frac{1}{(\sqrt{4})^3} = \frac{1}{8}$

2.) Simplify

a) $7^{\frac{2}{3}} * 7^{\frac{1}{2}} = 7^{4/6 + 3/6} = 7^{7/6}$

b) $(7^{\frac{2}{3}})^{\frac{1}{2}}$ $7^{\frac{2}{3}\cdot\frac{1}{2}} = 7^{1/3}$

b) $\frac{x^{-1}y^{\frac{1}{4}}}{x^{\frac{1}{4}}y^2}$ $\frac{y^{1/4}}{x \cdot x^{1/4} \cdot y^2} = y^{1/4-2} \cdot x^{-1-1/4} = y^{-7/4} \cdot x^{-5/4} = \frac{1}{y^{7/4} x^{5/4}} \cdot \frac{y^{1/4}}{y^{1/4}} \cdot \frac{x^{3/4}}{x^{3/4}} = \frac{x^{3/4} y^{1/4}}{y^2 x^2}$

7.3 Operations with Functions

1.) If $f(x) = 2x - 1$ and $g(x) = x - 5$, find:

a) $f(x) - g(x)$

$2x - 1 - (x - 5)$

$2x - x - 1 + 5$

$x + 4$

b) $f(x) + g(x)$

$2x - 1 + x - 5$

$3x - 6$

c) $f(x) \cdot g(x)$

$(2x - 1)(x - 5)$

$2x^2 - 11x + 5$

d) $f(g(x))$

$2(x - 5) - 1$

$2x - 10 - 1$

$2x - 11$

2.) Let $f(x) = 2x^2; g(x) = 3x^3$. Find $f(g(x))$

$2(3x^3)^2 = 2 \cdot 3^2 \cdot x^6 = 18x^6$

7.4 Finding an Inverse Function

3.) Find the inverse of $f(x) = 3x - 15$

$y = 3x - 15$

$x = 3y - 15$

$\frac{x+15}{3} = \frac{3y}{3}$

$f^{-1}(x) = \frac{x+15}{3}$ or $= \frac{1}{3}x + 5$

4.) Find the inverse of $f(x) = x^2 - 1$

$y = x^2 - 1$

$x = y^2 - 1$

$\sqrt{x+1} = y^2$

$f^{-1}(x) = \sqrt{x+1}$

7.5 Graphing Radical Functions

5.) Describe the transformation for each equation, then state the domain and range without graphing.

a.) $y = -\frac{1}{5}\sqrt{x+5} - 7$

Transformations: 5 left, 7 down

Domain: $x \geq -5$ Range: $y \leq -7$

b.) $y = 6\sqrt{x} + 3$

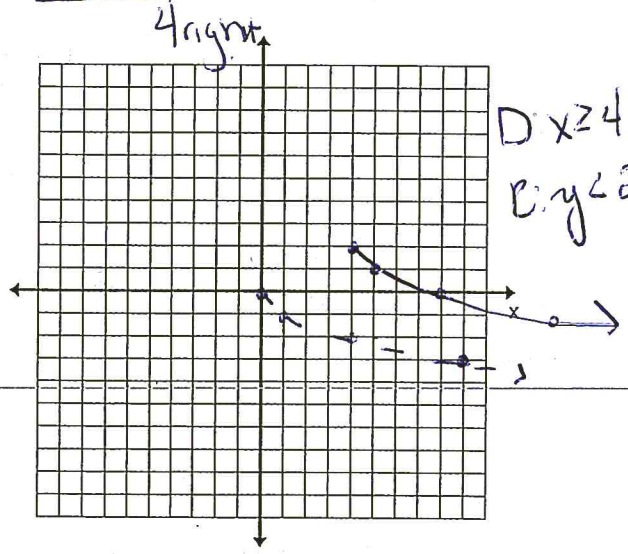
Transformations: 3 up

Domain: $x \geq 0$ Range: $y \geq 3$

6.) Graph each function on the grid provided.

a.) $y = -\sqrt{x-4} + 2$ 2up 4right

x	$a\sqrt{x}$
x	$-\sqrt{x}$
0	0
1	-1
4	-2
9	-3

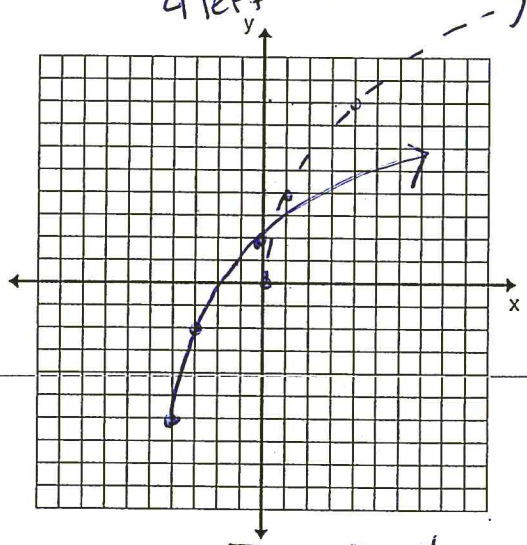


D: $x \geq 4$

R: $y \leq 2$

b.) $y = 4\sqrt{x+4} - 6$ 4 left 6 down

x	$a\sqrt{x}$
x	$4\sqrt{x}$
0	0
1	4
4	8
9	12

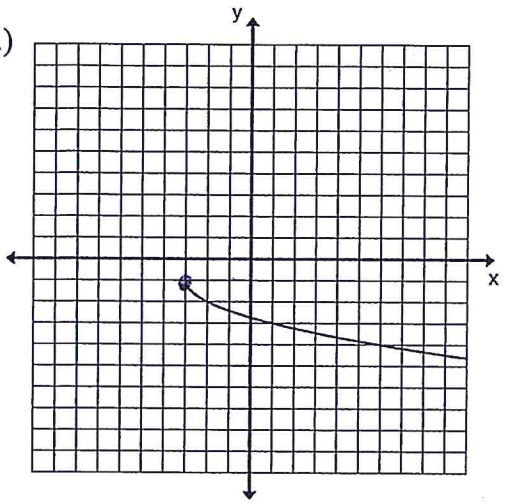


D: $x \geq -4$

R: $y \geq 6$

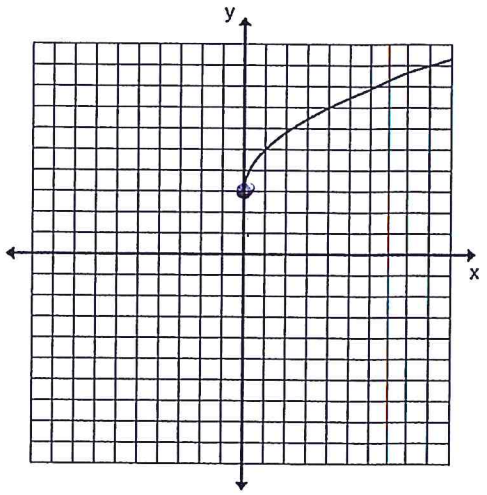
7.) State the domain and range of each graph

a.)



Domain: $x \geq -3$ Range: $y < -1$

b.)



Domain: $x \geq 0$ Range: $y > 3$

7.6 Solving Radical Equations

Solve each equation completely. Simplify any radicals; there should be no decimals in answer.

8.) $\sqrt{x-1}+4=9$.

$-4 \quad -4$

$(\sqrt{x-1})^2 = 5^2$

$x-1=25$

$+1 \quad +1$

$x=26$

9.) $(\sqrt{x-1})^2 = (x-3)^2$

$x-1 = x^2-6x+9$

$-x \qquad -x$

$-1 = x^2-7x+9$

$+1 \qquad +1$

$0 = x^2-7x+10$

$0=(x-5)(x-2)$

$x=5, 2$ extraneous

10.) $3(x+4)^4 = 243$

$\sqrt[4]{(x+4)^4} = \sqrt[4]{81}$

$x+4 = \pm 3$

$-4 \quad -4$

$x=-1, -7$

11.) $292 = 2x^3 - 140$

$+140 \qquad +140$

$\frac{432}{2} = \frac{2x^3}{2}$

$\sqrt[3]{216} = \sqrt[3]{x^3}$

$x=6$

12.) $\sqrt{3x+1} - \sqrt{x-5} = 0$

$(\sqrt{3x+1})^2 = (\sqrt{x-5})^2$

$3x+1 = x-5$

$-x \qquad -x$

$2x+1 = -5$

$-1 \quad -1$

$\frac{2x}{2} = \frac{-6}{2}$

$x=-3$ $\emptyset$

13.) $5x^{\frac{2}{3}} = 80$

$\frac{5}{5} \quad \frac{}{5}$

$(x^{2/3})^{3/2} = (16)^{3/2}$

$x = 16^{3/2}$

$x = (\sqrt{16})^3$

$x = (\pm 4)^3$

$x = \pm 64$

8.1/8.2 Exponential Growth and Decay

14.) a)Which function models exponential growth?

(A) $y = 3x$ linear (B) $y = 3^x$ $b>1$ (C) $y = \left(\frac{1}{3}\right)^x$ decay, $b<1$ (D) $y = x^3$ cubic

b)Which function models exponential decay?

(A) $y = -2x$ linear (B) $y = \left(\frac{1}{2}\right)^x$ (C) $y = (-2)^x$ (D) $y = x^{-2}$

15.) College tuition increases by about 8% per year. If York College of Pennsylvania costs $8,600 in 2002, how much will tuition be in 2010?

Equation: $y = 8{,}600(1+.08)^t$

Amount in 2010: $t = 8$ $y = 8{,}600(1+.08)^8 = \$15{,}918$

16.) You buy a used car for $12,000. It depreciates at the rate of 12% per year. Find the value of the car after 4 years.

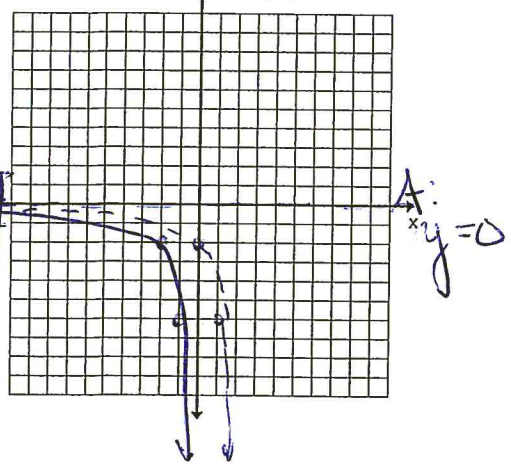
Equation: $y = 12{,}000(1-.12)^t$

Amount after 4 years: $t = 4$ $y = 12{,}000(1-.12)^4 = \$7{,}196.34$

17.) Graph each function. Label 2 points and the asymptote.

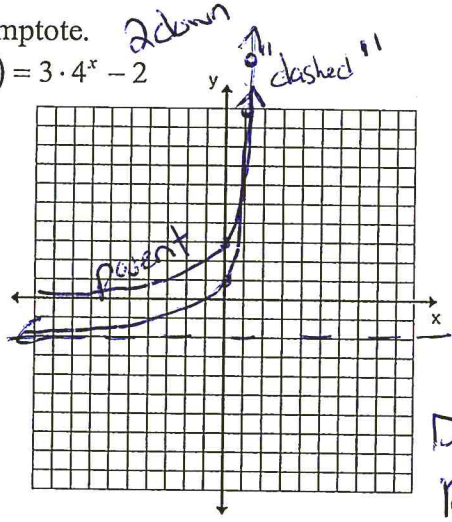
a) $f(x) = -2 \cdot 3^{x+2}$ 2 left

(0,4) (1,18) (0,-2) (1,-6) (-2,-2) (-1,-6)



D: $\mathbb{R}$ R: $y<0$

b) $f(x) = 3 \cdot 4^x - 2$ 2 down



(0,4) (1,48) (0,3) (1,12) (0,1) (1,10)

D: $\mathbb{R}$ R: $y>-2$

8.3 The number *e*

18.) The number e is approximately: (circle one)

(A) 3.14 (B) 1.69 (C) 6.55 (D) 2.72

19.) Simplify each expression. Do NOT evaluate.

a) $\frac{2e^8}{6e^2}$ $\frac{e^6}{3}$ b) $(4e^3)^2$ $4^2 e^6 = 16e^6$

8.4 Logarithmic Functions

20.) Change $5^x = 140$ into logarithmic form. $\log_5 140 = x$

21.) Change $\ln(10) = x - 1$ into exponential form. $e^{x-1} = 10$

22.) Evaluate each log.

a) $\log_2(4) = 2$ b) $\log_{10} 100 = 2$ c) $\log_5\left(\frac{1}{5}\right) = -1$

~~23.~~) Find the inverse of each function. Skip

a) $y = \ln(x + 1)$ b) $y = 6^{x-3} + 4$

24.) Expand: $\log_2 \frac{4x^2}{y}$ $\log_2 4x^2 - \log_2 y = \log_2 4 + 2\log_2 x - \log_2 y$

25.) Condense: $3\ln 2 - 2\ln x + \ln y$ $\ln 2^3 - \ln x^2 + \ln y = \ln \frac{8}{x^2} + \ln y = \ln \frac{8y}{x^2}$

8.6 Solving Exponential Equations

Solve each equation.

26.) $10^{x-3} = 100^{4x-5}$

$x - 3 = 2(4x - 5)$

$x - 3 = 8x - 10$

$-x \quad -x$

$-3 = 7x - 16$

$+16 \quad +16$

$\frac{7}{7} = \frac{7x}{7}$ $x = 1$

27.) $3^{0.1x} - 4 = 5$.

$+4 \quad +4$

$3^{.01x} = 9$

$3^{.01x} = 3^2$

$\frac{.01x}{.01} = \frac{2}{.01}$ $x = 200$

28.) $\log_3 x = -2$

$3^{-2} = x$ $x = \frac{1}{3^2} = \frac{1}{9}$

29.) $\log(3x - 8) = \log(x - 2)$

$3x - 8 = x - 2$

$-x \quad -x$

$2x - 8 = -2$

$+8 \quad +8$

$\frac{2x}{2} = \frac{6}{2}$ $x = 3$

30.) $2\ln x - 6 = 4$

$+6 \quad +6$

$\frac{2\ln x}{2} = \frac{10}{2}$

$\ln x = 5$

$e^5 = x$ $x = 146.413$

31.) Kim needs to save $10,500 to buy her first car. So far she has $6,500 saved. If she earns 5.2% interest each year, how long will it take her to reach her goal?

$\frac{10,500}{6,500} = \frac{6,500(1 + .052)^t}{6,500}$

$1.615 = (1.052)^t$

$\log_{1.052} 1.615 = t$

$t \approx 9.5$ years

9.1 and 9.2

32) The variables x and y vary inversely.

a) Write an equation relating x and y, using k as the constant of variation
b) Find k when x = 6 and y = 9
c) Find y when x = 3

$y = \frac{k}{x}$ $6 \cdot 9 = \frac{k}{6} \cdot 6$ $K = 54$ $y = \frac{54}{x}$ $y = \frac{54}{3}$

$y = 18$

33) The variable z varies jointly with x and y

a) Write an equation realting x, y, and z using k as the constant of variation
b) Find k when x =2, y = 4 , ans z = 6
c) Find z when x = 2 and y = -3

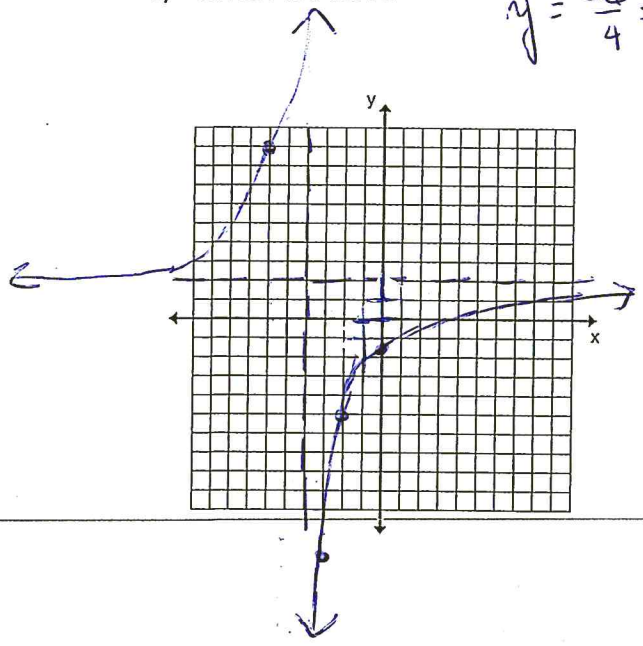
$z = kxy$ $6 = k \cdot 2 \cdot 4$ $\frac{6}{8} = \frac{k \cdot 8}{8}$ $k = \frac{3}{4}$ $z = \frac{3}{4}xy$

$z = \frac{3}{4} \cdot \frac{-3}{2}$

$z = \frac{-9}{2}$

34) For the function $y = \frac{2x-6}{1x+4}$, $x \neq -4$

a) Give the vertical asymptote $x = -4$
b) Give the horizontal asymptote $y = 2$
c) Give the y intercept $y = \frac{-6}{4} = \frac{-3}{2}$ $(0, -\frac{3}{2})$
d) Sketch the curve



x	-6	-5	-4	-3	-2
y	9	+16	/	-12	-5

$\frac{-4-6}{2} = -5$

$\frac{-12-6}{-2} = \frac{-18}{-2} = 9$ $\frac{2(-5)-6}{-1} = \frac{+16}{+1}$ $\frac{2(-3)-6}{1} = \frac{-12}{1}$

9.4-9.6

Simply

35.) $\frac{x^2-8x-9}{x^2-1}$

$\frac{(x-9)(x+1)}{(x-1)(x+1)}$

$\frac{x-9}{x-1}$

36.) $\frac{3x^2-12}{5x-10} * \frac{1}{2x+4}$

$\frac{3(x^2-4)}{5(x-2)} \cdot \frac{1}{2(x+2)}$

$\frac{3(x-2)(x+2)}{5(x-2)} \cdot \frac{1}{2(x+2)} = \frac{3}{10}$

$5(x^2-4)$

37.) $\frac{5x^2-20}{25x^2} \div \frac{x^2+6x+8}{x^2+10x+24}$

$\frac{5(x-2)(x+2)}{25x^2} \cdot \frac{(x+6)(x+4)}{(x+4)(x+2)}$

$= \frac{5(x-2)(x+6)}{25x^2}$ $x^2+4x-12$ $= \frac{5x^2+20x-60}{25x^2}$

38.) $\frac{x}{x^2-x-30} - \frac{1}{x+5}$ $\frac{(x-6)}{(x-6)}$

$(x-6)(x+5)$

$\frac{x-(x-6)}{(x-6)(x+5)} = \frac{6}{(x-6)(x+5)}$

$\frac{x}{x}$

39.) $\frac{x}{x^2-9} + \frac{3}{x(x-3)}$ $\frac{(x+3)}{(x+3)}$

$(x-3)(x+3)$

$\frac{x^2+3(x+3)}{x(x-3)(x+3)}$

$\frac{x^2+3x+9}{x(x-3)(x+3)}$

40.) $\left(\frac{3x}{x-2} = 1 + \frac{6}{x-2}\right) x-2$

$3x = x-2+6$

$-x \quad -x$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$ $\quad \emptyset$

6.3 and 6.5 Operations with Polynomials

41.) Simplify each expression.

a) $5(2x-3)^2$

$5(4x^2-12x+9)$

$20x^2-60x+45$

b.) $10-4(3x^2-6x+2)$

$10-12x^2+24x-8$

$-12x^2+24x+2$

c) $x(x^2-1)-3(2x+5)+(5x^3+4x-4)$

$x^3-x-6x-15+5x^3+4x-4$

$6x^3-3x-19$

42.) Divide (using any method):

a) $(6x^2+x-7)\div(2x+3)$

$$2x+3\overline{)\,6x^2+x-7}$$

$3x$

$-(6x^2+9x)$

$-8x-7$

$-(-8x-12)$

$+5$

$3x-4+\frac{5}{2x+3}$

b) $(x^2-4x+3)\div(x-2)$

$$2\;\big|\;1\;\;-4\;\;3$$
$$\qquad\;\;2\;\;-4$$
$$\overline{\qquad\;1\;\;-2\;\;-1}$$

$x-2+\frac{-1}{x-2}$