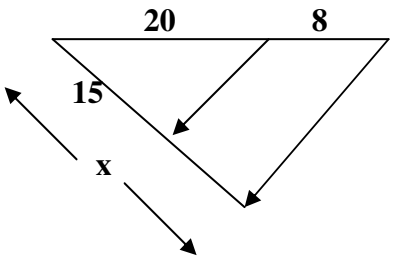
Notes for Lesson 7-6: Proportional Lengths

Triangle Proportionality Theorem:

If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.

Example: Given the following triangle, find two ways to solve for x



1st way: we can set up the following comparison to solve for x

$$\frac{\text{whole side}}{\text{top}} = \frac{\text{whole side}}{\text{top}}$$

$$\frac{x}{15} = \frac{28}{20}$$

2nd way: we can also set up the following comparison to solve for x

$$\frac{\text{top}}{\text{bottom}} = \frac{\text{top}}{\text{bottom}}$$

$$\frac{15}{x - 15} = \frac{20}{8}$$

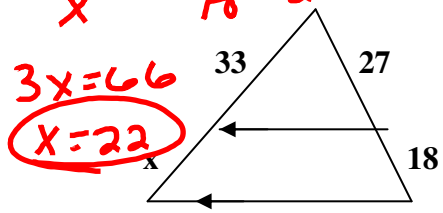
Find the value of x.

1)

$\frac{33}{x} = \frac{27}{18}\ \frac{3}{2}$

$3x = 66$

$x = 22$

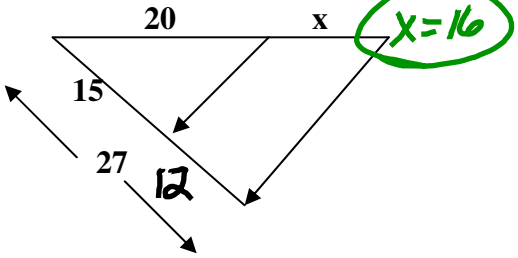


2)

$\frac{5}{4}\ \frac{15}{12} = \frac{20}{x}$

$5x = 80$

$x = 16$

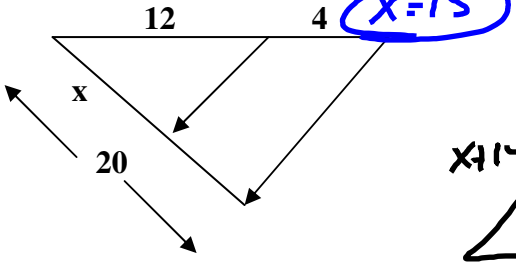


3)

$\frac{x}{20} = \frac{12}{16}\ \frac{3}{4}$

$4x = 60$

$x = 15$



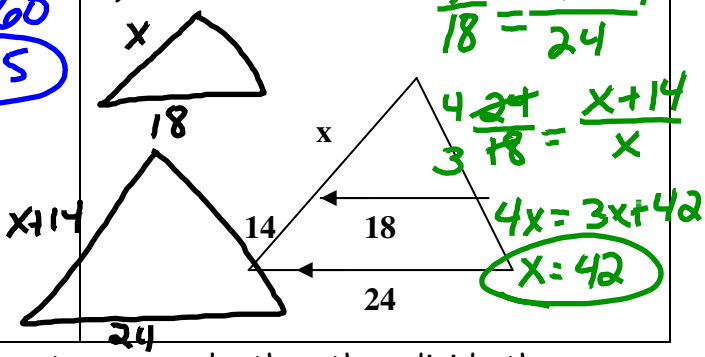
4)

$\frac{x}{18} = \frac{x+14}{24}$

$\frac{4}{3}\ \frac{24}{18} = \frac{x+14}{x}$

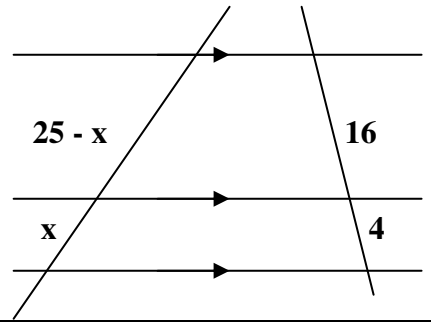
$4x = 3x + 42$

$x = 42$



If three parallel lines intersect two transversals, then they divide the transversals proportionally.

Example: Use the diagram to set up a proportion to solve for x.



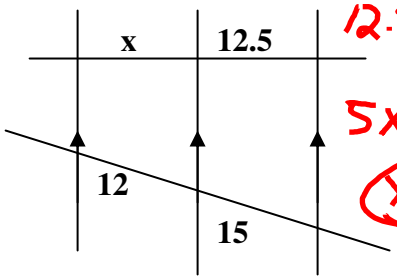
We set up the proportion in the following way:

$$\frac{\text{top left}}{\text{bottom left}} = \frac{\text{top right}}{\text{bottom right}}$$

$$\frac{25 - x}{x} = \frac{16}{4}$$

Find the value of x.

1)

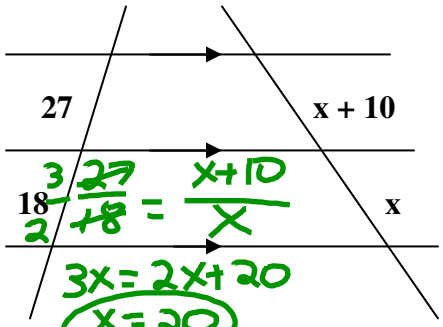


$\frac{x}{12.5} = \frac{124}{155}$

$5x = 50$

$x = 10$

2)



$\frac{27}{18} = \frac{x+10}{x}$

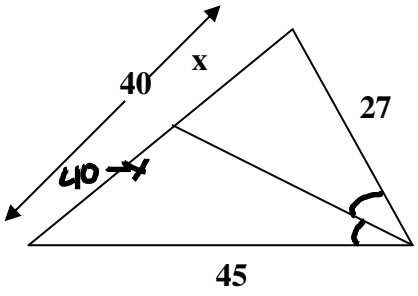
$3x = 2x + 20$

$x = 20$

Triangle Angle-Bisector Theorem:

If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two.

Example: Set up a proportion to solve for x.

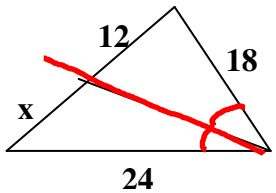


We can set the proportion up the following way:

$$\frac{\text{bottom}}{\text{bottom left}} = \frac{\text{top}}{\text{top left}}$$

$$\frac{45}{40 - x} = \frac{27}{x}$$

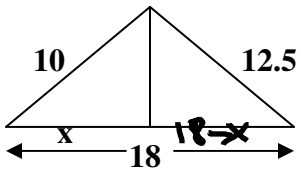
Solve for x.



$\frac{x}{24} = \frac{12}{18} \frac{2}{3}$

$3x = 48$

$x = 16$



$\frac{10}{x} = \frac{12.5}{18 - x}$

$12.5x = 180 - 10x$

$180 = 22.5x$

$x = 8$