Algebra IB

Remediation Workbook 2014-2015

Student Name: ____

Property of Dallastown Area School District

Unit 1

Functions and Relations

Objectives:

1. Define relation, function, domain, and range
2. Be able to determine if a relation is a function given ordered pairs or a table
3. Be able to determine if a relation is a function given a graph (including 2 part piecewise)
4. Be able to identify the domain and range of a relation given ordered pairs or a table
5. Be able to identify the domain and range of a relation given a graph (should be able to use inequality signs and interval notation; include 2 part piecewise graphs as well)
6. Analyze a set of data for the existence of a pattern by completing a table or a sequence
7. Be able to express a pattern as an equation given a table
8. Be able to represent a sequence as an expression

Objectives I must do for Test B: (circle ones you must do)

1 2 3 4 5 6 7 8

Determining a Function given Ordered Pairs

A **function** is a pairing between two sets of numbers in which each element of the first set is paired with exactly one element of the second set.

Examples

1. $\{(2,3),\ (3,4),\ (4,5),\ (5,6)\}$ is a **function** because each first element (*x*-coordinate or input) is paired with a different second element (*y*-coordinate or output).

2. $\{(2,3),\ (3,4),\ (4,5),\ (3,6)\}$ is **not a function** because the first element 3 is paired with 4 and also with 6. In order to have a function, 3 can only be paired with one second element.

Try These

Determine if the following relations are functions.

1. $\{(4,3),\ (-2,10),\ (5,-6),\ (10,7)\}$ — 1. ______________________

2. $\{(-3,-6),\ (-5,10),\ (-1,2),\ (0,0)\}$ — 2. ______________________

3. $\{(2,7),\ (3,7),\ (5,7),\ (6,7)\}$ — 3. ______________________

4. $\{(7,2),\ (7,3),\ (7,4),\ (7,5)\}$ — 4. ______________________

5. $\{(-5,3),\ (6,5),\ (3,2),\ (10,3)\}$ — 5. ______________________

6. $\{(6,4),\ (-5,2),\ (6,7),\ (-8,8)\}$ — 6. ______________________

7. $\{(11,5),\ (2,7),\ (-3,8),\ (-3,10)\}$ — 7. ______________________

8. $\{(9,4),\ (3,2),\ (-6,4),\ (8,7)\}$ — 8. ______________________

9. $\{(-7,4),\ (8,12),\ (9,12),\ (6,13)\}$ — 9. ______________________

10. $\{(8,6),\ (-5,2),\ (0,6),\ (-5,1)\}$ — 10. ______________________

Determining a Function given a Table

Examples

1.

x	y
1	–4
5	–3
8	–2
9	–2

This is a **function** because each first element (x-coordinate or input) is paired with a different second element (y-coordinate or output).

2.

x	y
–7	0
–8	–1
–9	2
–7	–3

This is **not a function** because the first element –7 is paired with 0 and also with –3. In order to have a function, –7 can only be paired with one second element.

Try These

Determine if the following relations are functions.

1.

x	y
1	6
1	–6
2	8
2	–8

2.

x	y
1	4
2	4
3	4
4	4

3.

x	y
–4	2
–4	3
–4	4
–4	5

4.

x	y
3	2
5	4
7	5
3	7

5.

x	y
0	–5
2	6
9	–3
5	0

6.

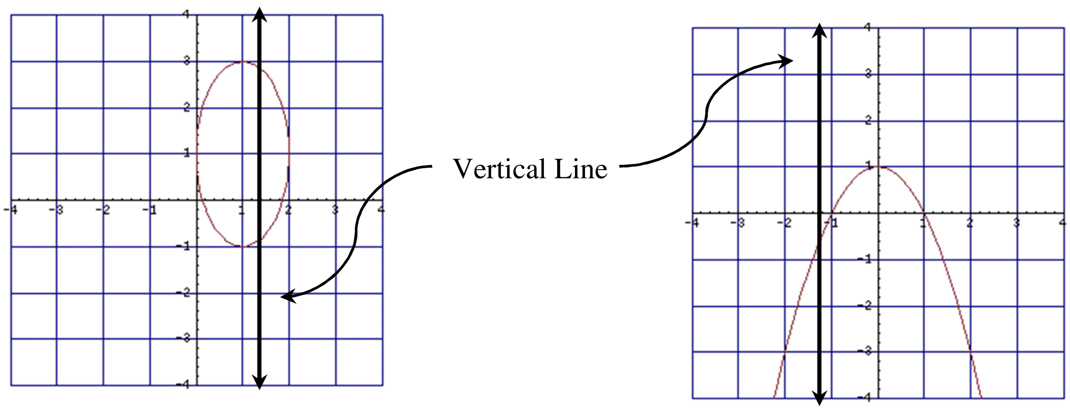
x	y
0	–6
–2	–4
4	–2
–6	–2

Determining if a Graph is a Function

To determine if a graph is a function or not a function, we use the **Vertical Line Test**. If a vertical line intersects the graph at more than one point, then the graph is not a function. A graph is a function if a vertical line does not intersects the graph at more than one point.

Examples

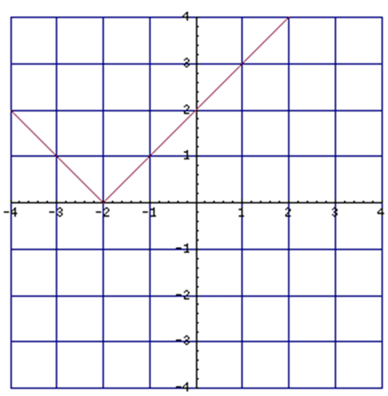
1. This graph is **not a function** because a vertical line intersects the graph at more than one point.
2. This graph is a **function** because any vertical line does not intersect the graph at more than one point.



Try These

Determine if the following graphs are functions or not functions.

1.



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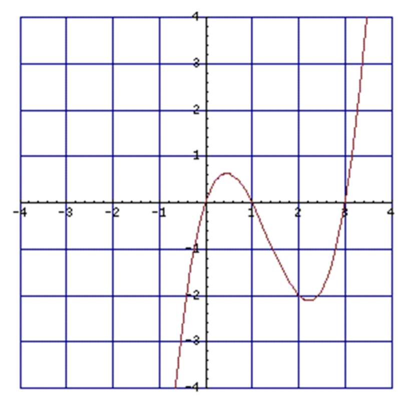

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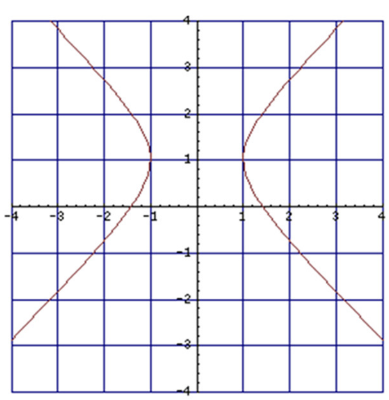

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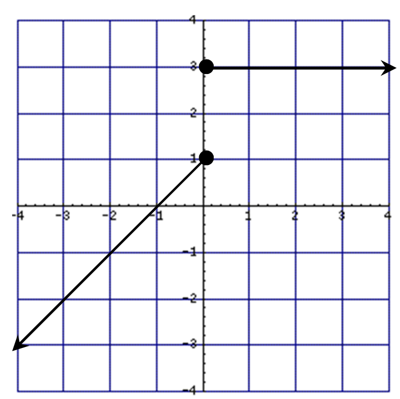

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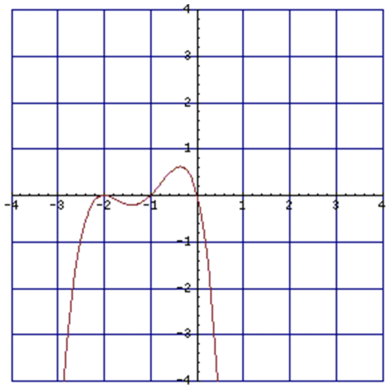

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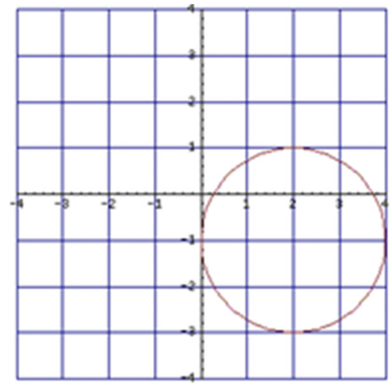

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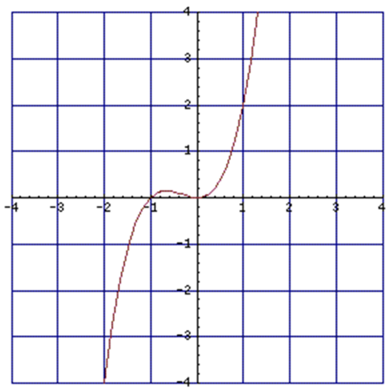

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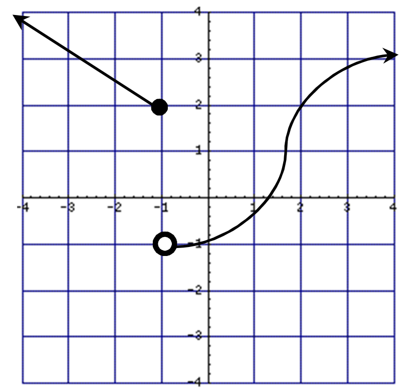

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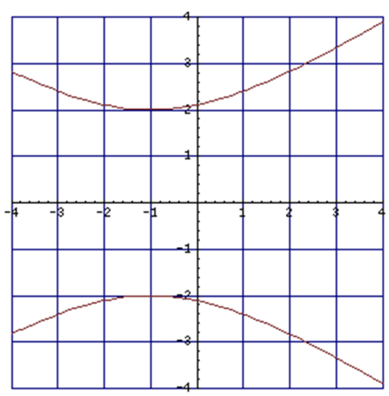

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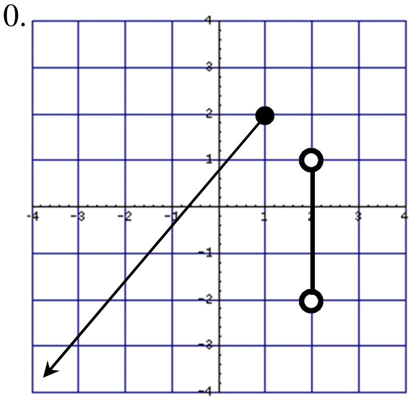

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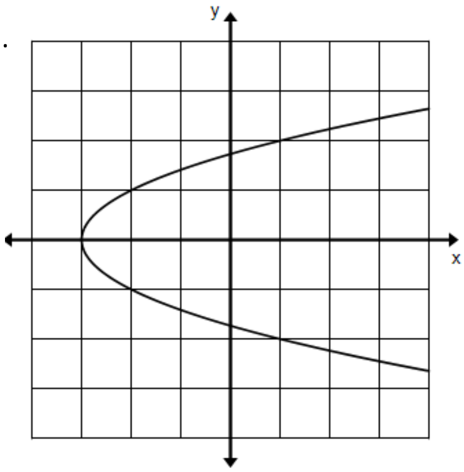

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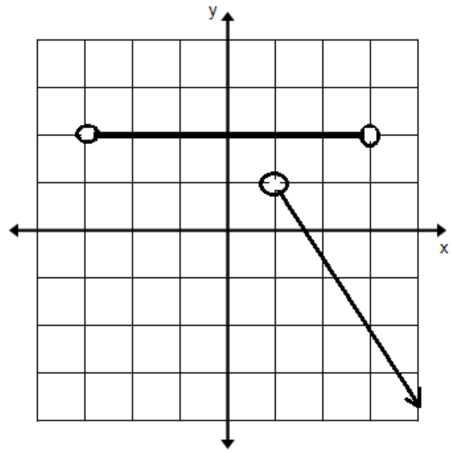

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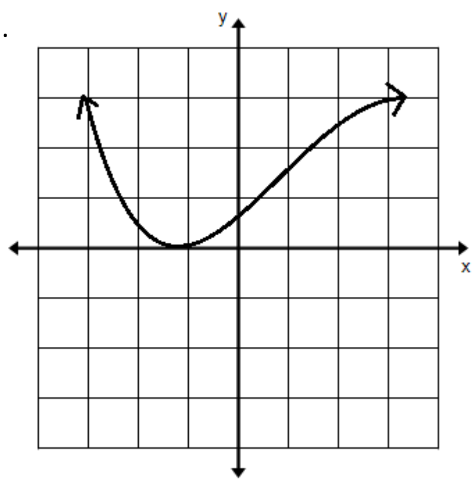

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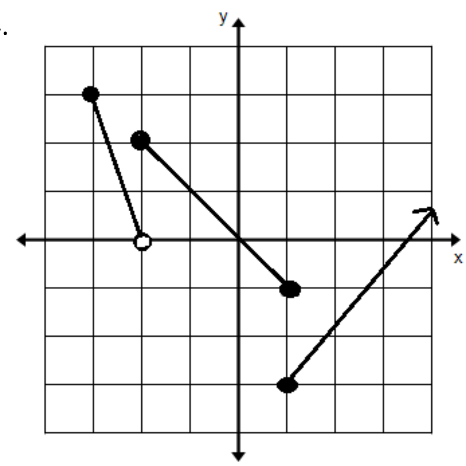

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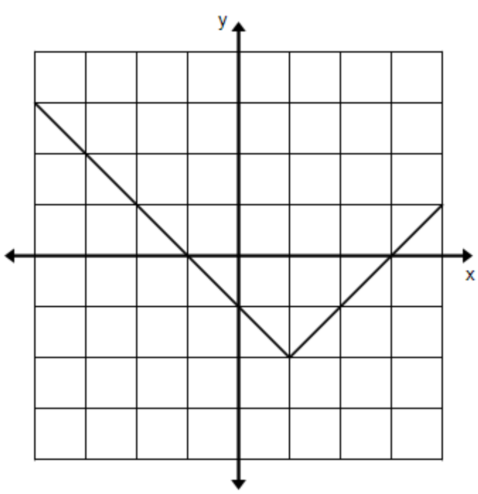


Finding Domain and Range given Ordered Pairs

The **domain** is the set of the first coordinates in a set of ordered pairs of a relation or function (usually the *x*-coordinate).

The **range** is the set of the second coordinates in a set of ordered pairs of a relation or function (usually the *y*-coordinate).

Examples

State the domain and range: $\{(\mathbf{2},\mathbf{3}),\ (\mathbf{3},\mathbf{4}),\ (\mathbf{4},\mathbf{5}),\ (\mathbf{5},\mathbf{6})\}$

The domain is all the *x*-coordinates. $D = \{2, 3, 4, 5\}$
The range is all the *y*-coordinates. $R = \{3, 4, 5, 6\}$

State the domain and range: $\{(\mathbf{2},-\mathbf{3}),\ (\mathbf{3},-\mathbf{3}),\ (\mathbf{4},-\mathbf{5}),\ (\mathbf{3},-\mathbf{5})\}$

The domain is all the *x*-coordinates. $D = \{2, 3, 4\}$
The range is all the *y*-coordinates. $R = \{-5, -3\}$

Try These

State the domain and range for each relation.

1. $\{(4, 3),\ (-2, 10),\ (5, -6),\ (10, 7)\}$ 1. ______________________ ______________________

2. $\{(-3, -6),\ (-5, 10),\ (-1, 2),\ (0, 0)\}$ 2. ______________________ ______________________

3. $\{(-7, 4),\ (8, 12),\ (9, 12),\ (6, 13)\}$ 3. ______________________ ______________________

4. $\{(7, 2),\ (7, 3),\ (7, 4),\ (7, 5)\}$ 4. ______________________ ______________________

5. $\{(-5, 3),\ (6, 5),\ (3, 2),\ (10, 3)\}$ 5. ______________________ ______________________

6. $\{(6, 4),\ (-5, 2),\ (6, 7),\ (-8, 8)\}$ 6. ______________________ ______________________

Finding Domain and Range given a Table

Examples

State the domain and range for each relation.

x	y
1	–4
5	–3
8	–2
9	–2

Domain is the x-coordinates.
$D = \{1, 5, 8, 9\}$
Range is the y-coordinates.
$R = \{-4, -3, -2\}$

x	y
–7	0
–8	–1
–9	2
–7	–3

Domain is the x-coordinates.
$D = \{-9, -8, -7\}$
Range is the y-coordinates.
$R = \{-3, -1, 0, 2\}$

Try These

State the domain and range for each relation.

1.

x	y
1	6
1	–6
2	8
2	–8

____

____

2.

x	y
1	4
2	4
3	4
4	4

____

____

3.

x	y
–4	2
–4	3
–4	4
–4	5

____

____

4.

x	y
3	2
5	4
7	5
3	7

____

____

5.

x	y
0	–5
2	6
9	–3
5	0

____

____

6.

x	y
0	–6
–2	–4
4	–2
–6	–2

____

____

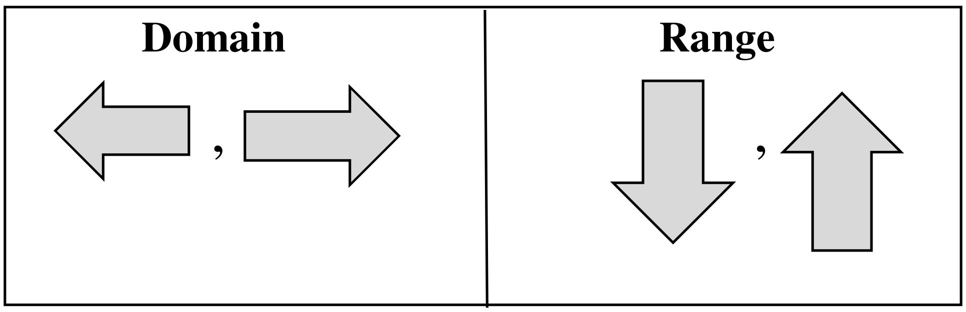
Finding Domain and Range from a Graph

Domain – input values, *x*-values. On a graph this means you are looking left and right

1. How far left does the graph go? How far right does the graph go?

Range – output values, *y*-values. On a graph this means you are looking up and down.

2. How far down does the graph go? How far up does the graph go?

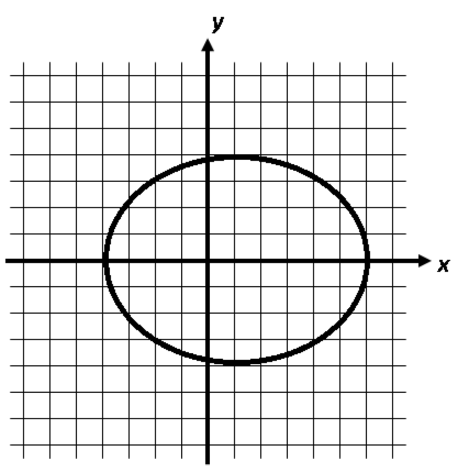


Interval Notation:

a. Use **(** and **)** for infinity (∞) and open end points (○) – this means there really isn't a point at that value
b. Use **[** and **]** for everything else – this means that there is a point at this value

Examples

1.)



__Domain:__

The most left is at -4, the most right is at 6. Both endpoints **are included** in the domain so we use [and]

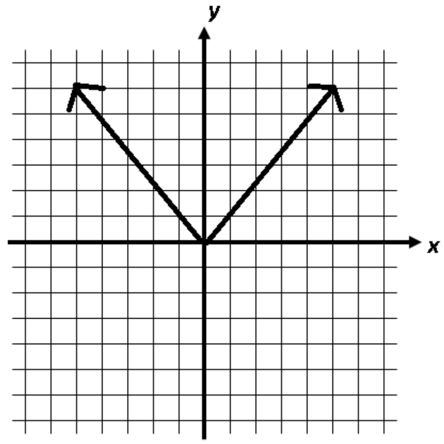
Domain: $[\mathbf{-4, 6}]$

__Range:__

The farthest down is at -4, and farthest up is at 4
Both endpoints **are included** in the domain so we use [and]

Range: $[\mathbf{-4, 4}]$

2.)



<u>Domain:</u>
The graph never stops going to the left $(-\infty)$ and never stops going to the right (∞)
We will use (and) since both endpoints are ∞

Domain: $(-\infty, \infty)$ **or R**

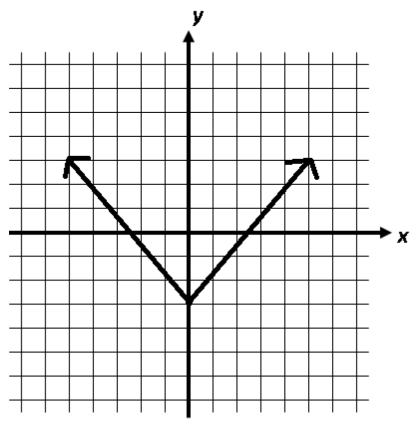
<u>Range:</u>
The farthest down is at 0, and the graph never stops going up (∞)
There is a point at 0 so we use [but we will use) for the ∞

Range: $[\mathbf{0}, \infty)$

Try Some

Find the domain and range of each graph. Write your answer in interval notation.

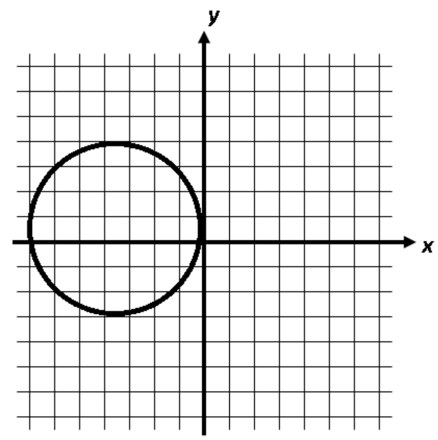
1.)



Domain:

Range:

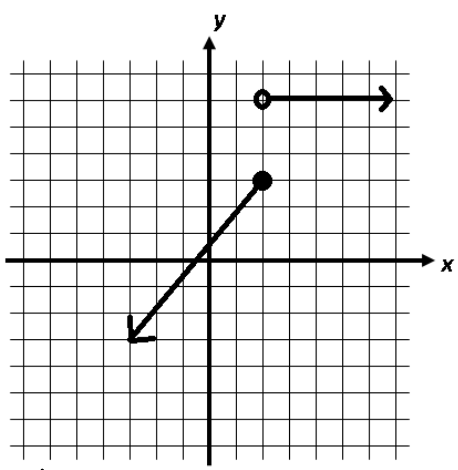
2.)



Domain:

Range:

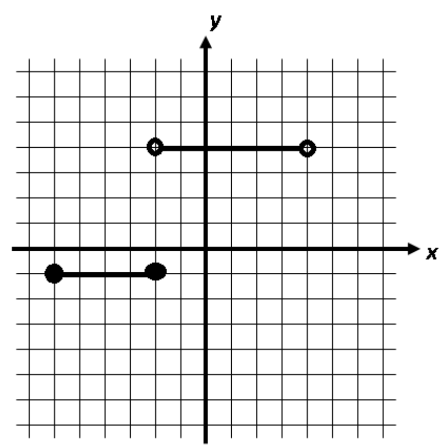
3.)



Domain:

Range:

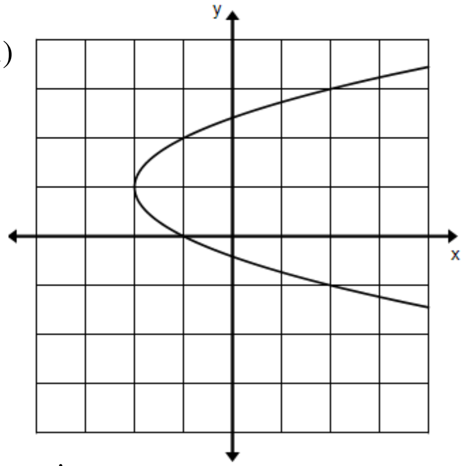
4.)



Domain:

Range:

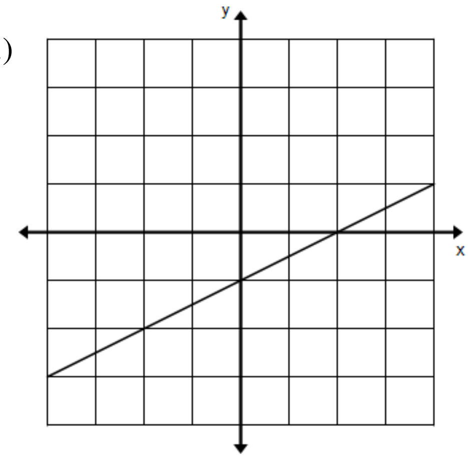
5.)



Domain:

Range:

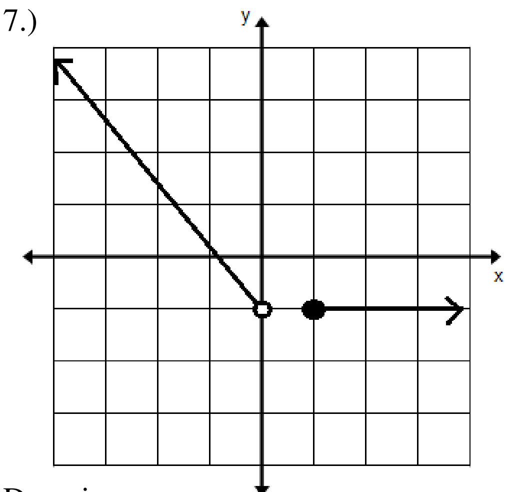
6.)



Domain:

Range:

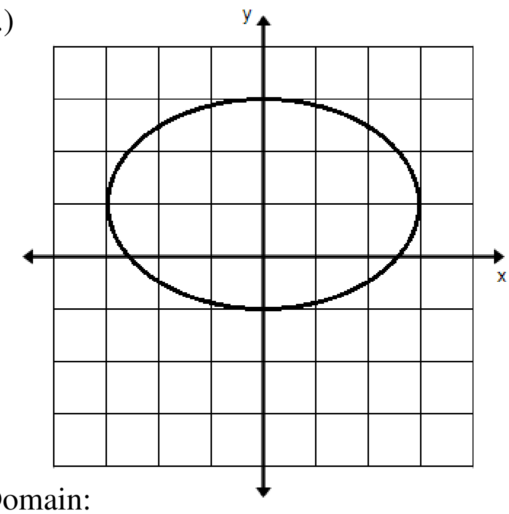
7.)



Domain:

Range:

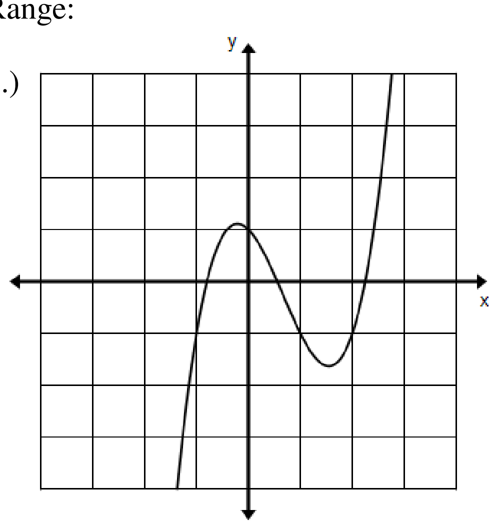
8.)



Domain:

Range:

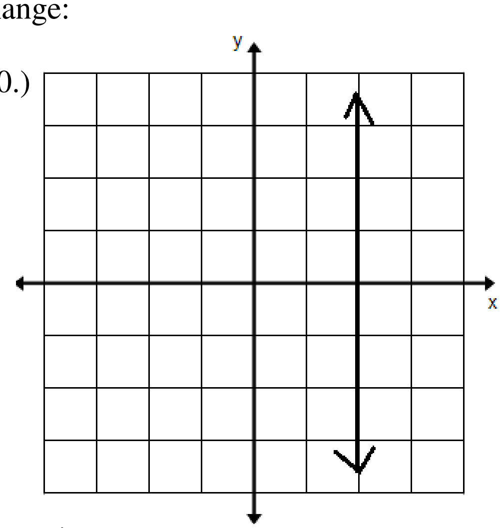
9.)



Domain:

Range:

10.)



Domain:

Range:

Analyze a Set of Data for the Existence of a Pattern by Completing a Table or a Sequence

Ways to identify a pattern:

1. Check to see if the same number is added, subtracted, multiplied, or divided
2. Check to see if there is a second level common sum or difference

Example A

1, 3, 5, 7, ___, ___, ___

The difference from one number to the next is +2, so to find the next three numbers, you add 2 to the previous number. Answers: 9, 11, 13

Example B

x	0	5	15	30	50	75
y	6	-12	24			

The numbers are going from positive to negative and are doubling. Therefore, you multiply by -2 to get the next number in the sequence. Answers: -48, 96, -192

Try some

Complete the table or sequence.

1.) $10,\ 15,\ 20,\ 25,$ ____, ____, ____

4.) $100,\ 50,\ 25,\ 12.5,$ ____, ____, ____

2.) $-45, -42, -39, -36,$ ____, ____, ____

5.) $2,\ 6,\ 18,\ 54,$ ____, ____, ____

3.) $8,\ 6,\ 4,\ 2,$ ____, ____, ____

6.) $12,\ 15,\ 19,\ 24,$ ____, ____, ____

7.)

x	10	20	30	40	50	60
y	3.5	3.6	3.7			

8.)

x	0	1	2	3	4	5
y	0.8	-3.2	12.8			

9.)

x	-4	-3	-2	-1	0	1
y	2.5	3.6	4.8			

10.)

x	0	2	4	6	8	10
y	-45	-48	-51			

11.)

x	-3	-2	-1	0	1	2
y	-10	-14	-19			

Express a Pattern as an Equation Given a Table

Being able to write an equation from a table is an important skill; it helps you find values that aren't in the table more easily.

<u>Steps in Writing an Equation from a Table</u>

1.) Use $y = mx + b$ (Slope-Intercept Form) → you must fill in numbers for *m* and *b*.

2.) Find *m* (Slope): $m = \frac{change\ in\ y}{change\ in\ x}$

3.) Find *b* (y-intercept) by finding the y value when $x = 0$. You may need to use the pattern to fill in more spots on the table.

4.) Substitute *m* and *b* into $y = mx + b$

Examples

1.) Write an equation to represent the table below:

x	0	1	2	3	4	5
y	12	15	18	21	24	27

Step 1 – Start with $y = mx + b$ → $y =$ __$x +$ ___

Step 2 – Find *m*: $m = \frac{\Delta y}{\Delta x} = \frac{15-12}{1-0} = \frac{3}{1} = 3$

Step 3 – Find *b*: Look at the table to where *x* = 0, we see that *y* = 12, this means $b = 12$

Step 4 – Fill in the values for *m* and *b*: $y = 3x + 12$ ← Final Equation

2.) Write an equation to represent the table below:

x	2	4	6	8	10	12
y	3	-2	-7	-12	-17	-22

Step 1 – Start with $y = mx + b$ → $y =$ __$x +$ ___

Step 2 – Find *m*: $m = \frac{\Delta y}{\Delta x} = \frac{-2-3}{4-2} = \frac{-5}{2}$

Step 3 – Find *b*: Since there is not a spot where *x* = 0 we have to continue the pattern

x	0	2
y	8	3

Work backwards to where *x* = 0, since *y* = 8 that means *b* = 8

Step 4 – Fill in values for *m* and *b*: $y = -\frac{5}{2}x + 8$ ← Final Equation

Try These

Write the equation for the given table. Write your final equation on the line provided.

1)

x	0	1	2	3	4	5
y	0.5	1	1.5	2	2.5	3

Equation: ______________________________

2)

x	-2	0	2	4	6	8
y	-12	-14	-16	-18	-20	-22

Equation:______________________________

3)

x	5	10	15	20	25	30
y	3	0	-3	-6	-9	-12

Equation:______________________________

4)

x	3	5	7	9	11	13
y	35	37	39	41	43	45

Equation:______________________________

5)

x	12	24	36	48	60	72
y	6	12	18	24	30	36

Equation:________________________________

6)

x	-8	-7	-6	-5	-4	-3
y	5	0	-5	-10	-15	-20

Equation:________________________________

7)

x	-3	-2	-1	0	1	2
y	-5	-3	-1	1	3	5

Equation:________________________________

8)

x	-4	-2	0	2	6	8
y	1	2	3	4	6	7

Equation:________________________________

Represent a Sequence as an Expression

A **sequence** is an ordered list. If a there is a pattern to the sequence we can represent the sequence with an expression. This is useful because we can use the expression to find any term (the n^{th} term) in the sequence very easily without having to write out all the numbers in the sequence.

Representing a sequence with an expression is very similar to writing an equation from a table. We will be using the $mx + b$ part to make our expression (an expression does not have an equal sign).

Example

Look at the following pattern: $8,\ 17,\ 26,\ 35,\ 44, \ldots$

Write an expression to that could be used to find the n^{th} term in this pattern.

Step 1: Label the numbers in the sequence as 1st term, 2nd term, 3rd term, etc

$\boldsymbol{n} =$	**1**	**2**	**3**	**4**	**5**	← **label above like this**
	8,	17,	26,	35,	44, …	

Step 2: Now that the terms are labeled, you are going to write the expression using the same steps as writing an equation from a table.

Use $mx + b$ but instead of *x* we will be using *n* since we are asked for the n^{th} term, so it will look like $mn + b$

Find *m* by taking 2nd term – 1st term: $m = 17 - 8 = 9$ → $\boldsymbol{m = 9}$

Find b by finding the initial term when $n = 0$, follow the pattern backwards:

0	**1**	**2**	**3**	**4**	**5**	→ $\boldsymbol{b = -1}$
$-1,$	8,	17,	26,	35,	44, …	

Step 3: Fill in your values for *m* and *b* into $mn + b$:

Final expression: $\boldsymbol{9n - 1}$

Check: If we put 5 in for *n* we should get 44 since the 5th term in the sequence is 44.

$9(5) - 1 = 45 - 1 = 44$ → This works!

Practice

Write an expression to that could be used to find the n^{th} term each pattern.

1.) $4,\ 9,\ 14,\ 19,\ 24, \ldots$

Expression: ________________________

2.) $9,\ 5,\ 1,\ -3,\ -7, \ldots$

Expression: ________________________

3.) $1.5,\ 3,\ 4.5,\ 6,\ 7.5, \ldots$

Expression: ________________________

4.) $32,\ 21,\ 10,\ -1,\ -12, \ldots$

Expression: ________________________

5.) $123,\ 181,\ 239,\ 297, \ldots$

Expression: ________________________

6.) $6,\ 30,\ 54,\ 78, \ldots$

Expression: ________________________

7.) Below are the scores that Jordan earned on his SAT exam.

$890, \quad 960, \quad 1030, \quad 1100, \quad 1170$

Jordan's scores follow a pattern. Write an expression that can be used to determine his SAT score after *n* times taking the test.

Expression: ________________________

8.) Tim's scores in the first 5 times he played a video game are listed below.

4,526 4,599 4,672 4,745 4,818

Tim's scores follow a pattern. Write an expression that can be used to determine his score after *n* times he played the video game.

Expression: ________________________

Unit 2

Linear Functions

Objectives:

1. Define linear equation, slope, x-intercept, y-intercept, slope-intercept form, and standard form
2. Identify the x-intercept and y-intercept given a graph
3. Identify the x-intercept and y-intercept given an equation
4. Determine the slope of a line given a graph
5. Determine the slope of a line given two points
6. Determine the slope of a line given an equation
7. Convert a linear equation to slope-intercept form
8. Convert a linear equation to standard form

Objectives I must do for Test B: (circle ones you must do)

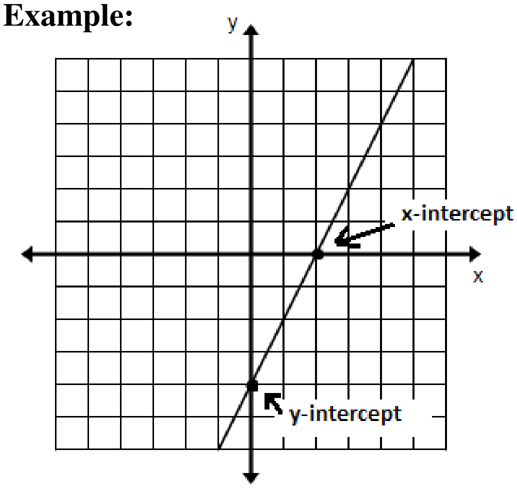
1 2 3 4 5 6 7 8

Identify the x-intercept and y-intercept Given a Graph

To find the ***x*-intercept** from a graph, locate the x axis that runs from the left to the right on the graph. Find where the line intersects this axis. The point will be (____, 0).

To find the ***y*-intercept** from a graph, locate the y-axis that runs from north to south on the graph. Find where the line intersects this axis. The point will be (0, ____).

Example:



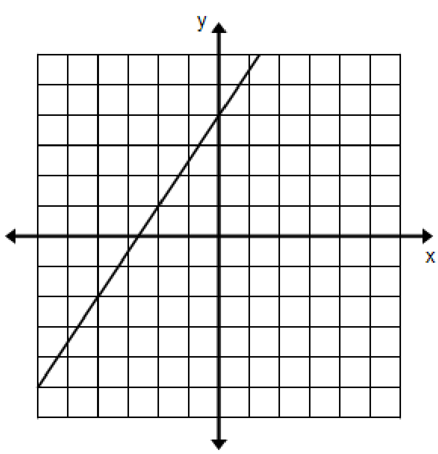
***x*-intercept**: $(2, 0)$

***y*-intercept**: $(0, -4)$

Try Some

Find the x-intercept and y-intercept of each graph.

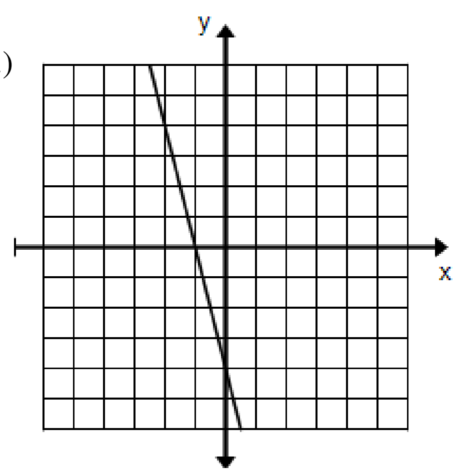
1.)



<u>x-int</u>:

<u>y-int</u>:

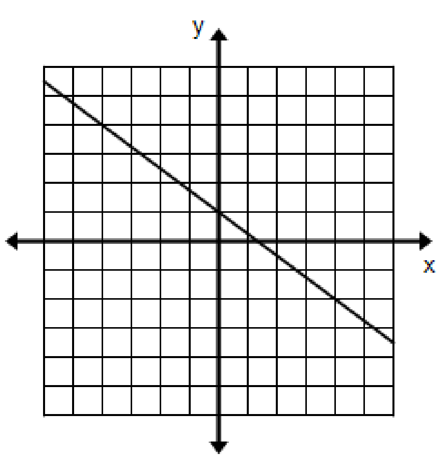
2.)



<u>x-int</u>:

<u>y-int</u>:

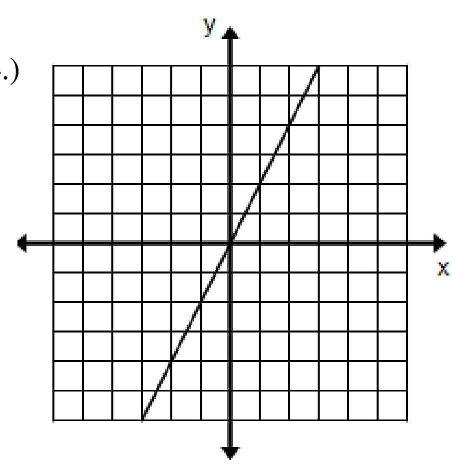
3.)



<u>x-int</u>:

<u>y-int</u>:

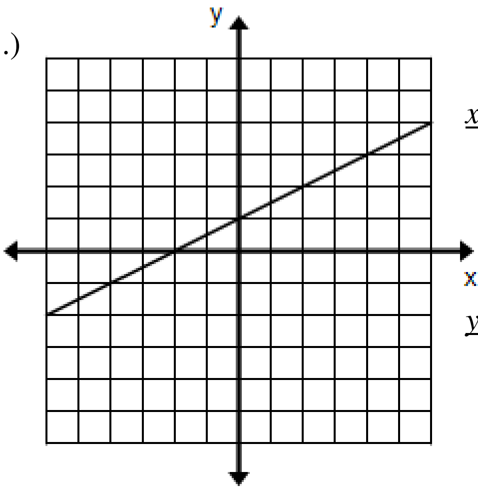
4.)



<u>x-int</u>:

<u>y-int</u>:

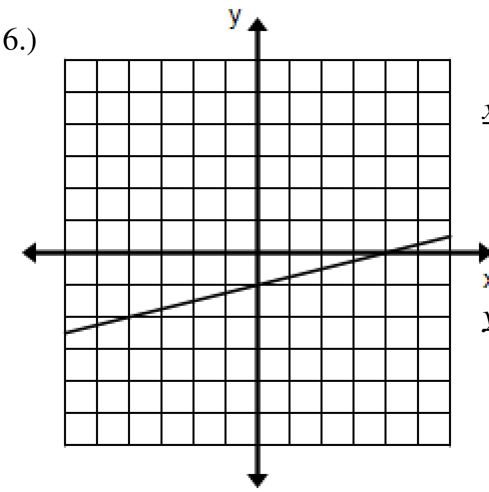
5.)



x-int:

y-int:

6.)



x-int:

y-int:

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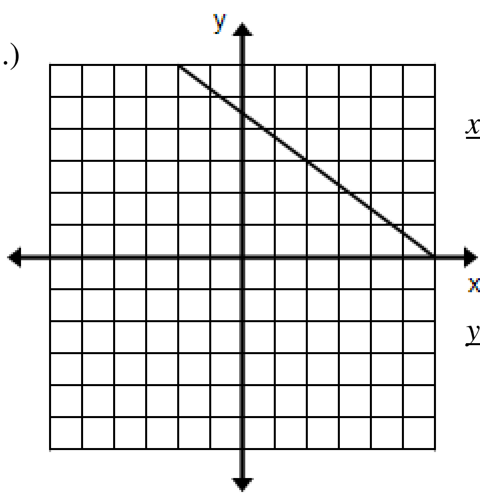
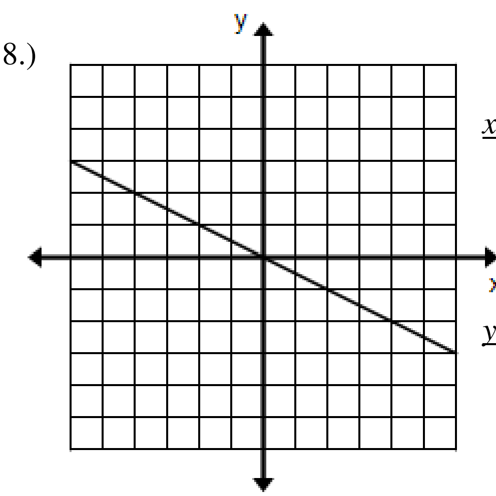


x-int:

y-int:

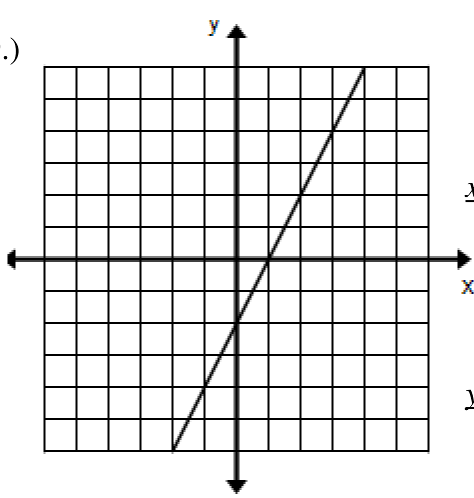
8.)



x-int:

y-int:

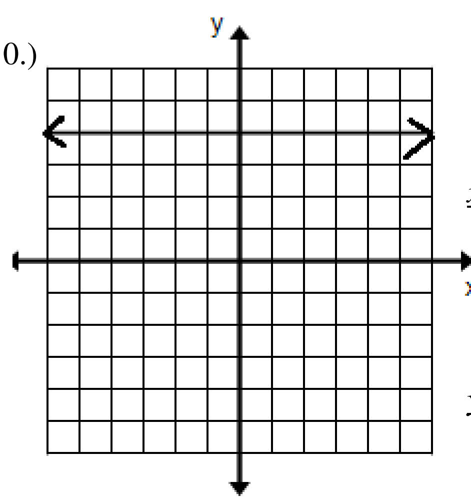
9.)



x-int:

y-int:

10.)



x-int:

y-int:

Identify the x-intercept and y-intercept Given an Equation

To find the $\boldsymbol{x}$**-intercept** from Standard Form, a zero will be used in place of the y value, since the x-intercept is always (___, 0). The y-value is always 0 for the x-intercept.

To find the $\boldsymbol{y}$**-intercept** from Standard Form, a zero will be used in place of the x value, since the y-intercept is always (0, ___). The x-value is always 0 for the y-intercept. NOTE: if the equation is in slope-intercept form ($y = mx + b$) then you can get the y-intercept right from the equation without doing any work. The y-intercept is the b-value.

****Whichever intercept you are finding, put a 0 in for the other variable**

Example

Find the x-intercept and y-intercept of the equation $2x - y = 6$

$\boldsymbol{x}$**-Intercep**t → put a 0 in for y, then solve for x

$2x - 0 = 6$

$2x = 6$

$x = 3$

$\boldsymbol{x}$**-intercept:** $(\mathbf{3}, \mathbf{0})$

$\boldsymbol{y}$**-Intercept** → put a 0 in for x, then solve for y

$2(0) - y = 6$

$0 - y = 6$

$-y = 6$

$y = -6$

$\boldsymbol{y}$**-intercept:** $(\mathbf{0}, -\mathbf{6})$

Try Some

Find the x-intercept and the y-intercept for each equation.

1. $4x - 3y = 12$

x-Int: _________ y-Int: ___________

2. $y = 3x + 6$

x-Int: _________ y-Int: ___________

3. $x - 3y = -3$

4. $y = \frac{2}{3}x - 6$

x-Int: _________ y-Int: ___________

x-Int: _________ y-Int: ___________

5. $2x + 5y = 10$

6. $y = -5x - 2$

x-Int: _________ y-Int: ___________

x-Int: _________ y-Int: ___________

7. $2x - y = -6$

8. $y = -\frac{7}{2}x + 3$

x-Int: _________ y-Int: ___________

x-Int: _________ y-Int: ___________

9. $4x + 3y = 4$

10. $y = 2x - 7$

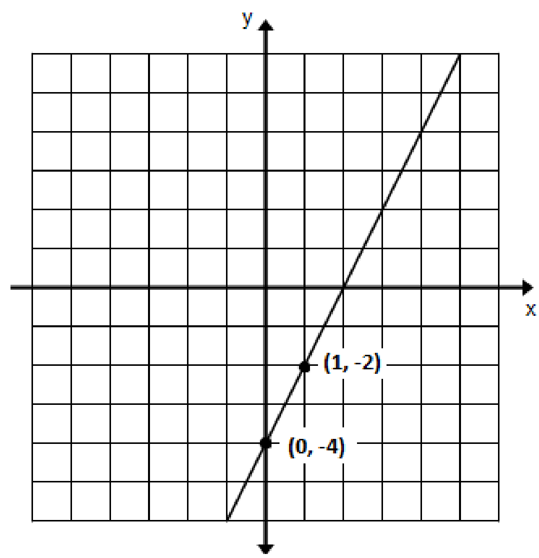
x-Int: _________ y-Int: ___________

x-Int: _________ y-Int: ___________

Determine the Slope of a Line Given a Graph

To find the **slope** of a line from a graph, two points must be found that the coordinates can be easily found. Coordinates that are easily found are from points that are found on the graph where the horizontal and vertical lines of the grid intersect. Once these two points are found, the slope can be determined one of two ways.

Method 1: Use the slope formula $\boldsymbol{m} = \frac{\boldsymbol{y_2 - y_1}}{\boldsymbol{x_2 - x_1}}$.



Step 1 – Find the coordinates of two points on the line

$(0, -4)$ and $(1, -2)$.

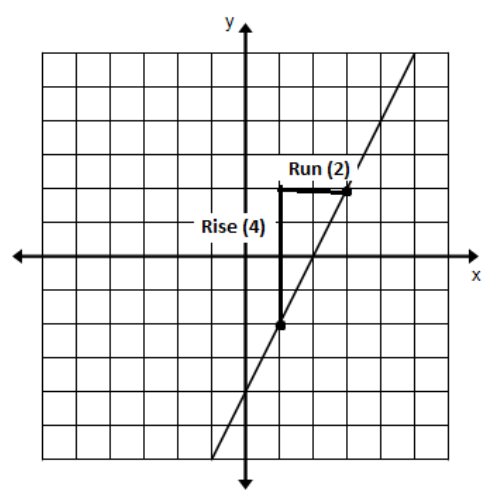
Step 2 – use the slope formula and plug in the values for

x and y using the points you found in step 1.

Using the formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-2-(-4)}{1-0} = \frac{2}{1} = 2$

Method 2: Use $\frac{\boldsymbol{rise}}{\boldsymbol{run}}$ on the graph. Remember that graphs that point up on the right side have positive slope and graphs that point down on the right side have negative slope.



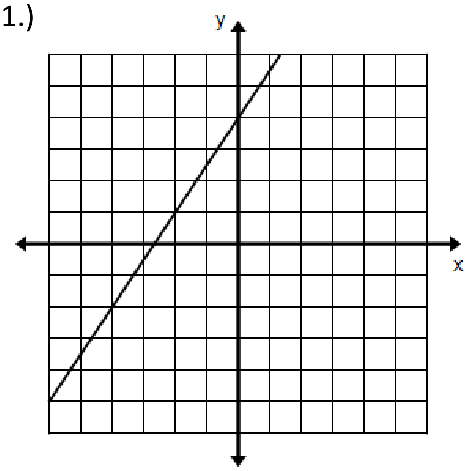
Start at $(1, -2)$ and go to $(3,2)$. The graph rises 4 blocks and runs 2 block. That makes $m = \frac{4}{2} = 2$.

**It does not matter which points you choose on the line, you will get the same slope.

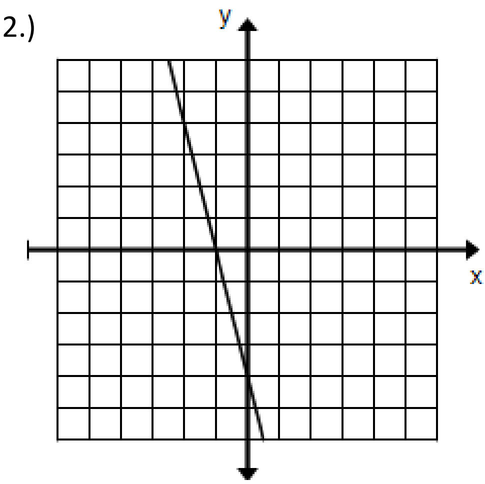
Try Some

Find the slope of each of these graphs using either Method 1 or Method 2.

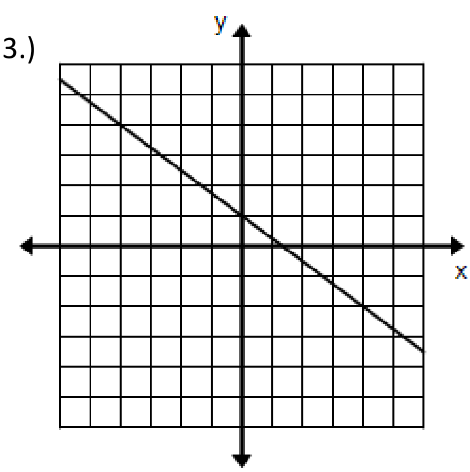
1.)



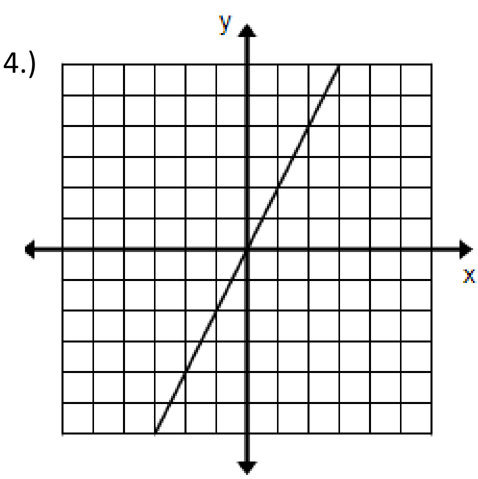
2.)



3.)



4.)



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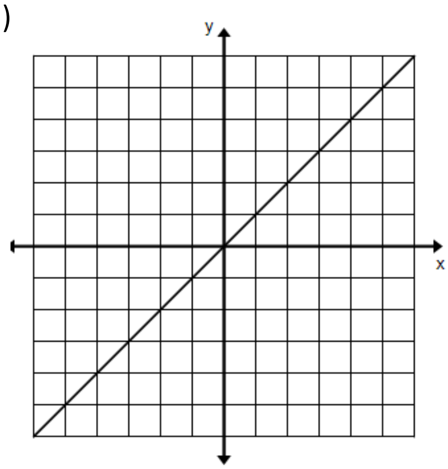
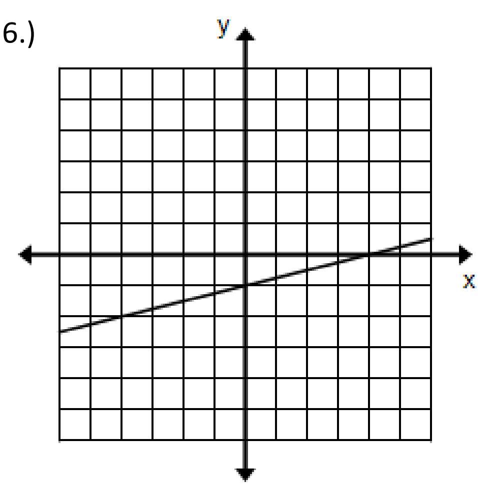
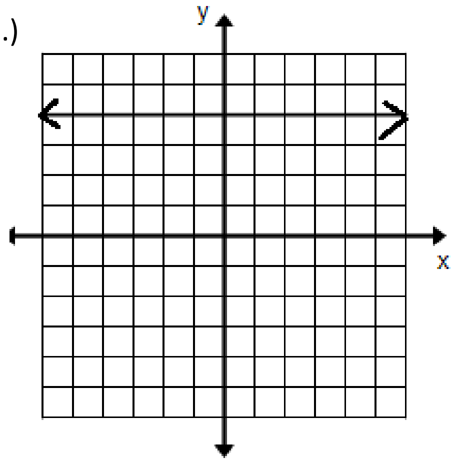


6.)



7.)



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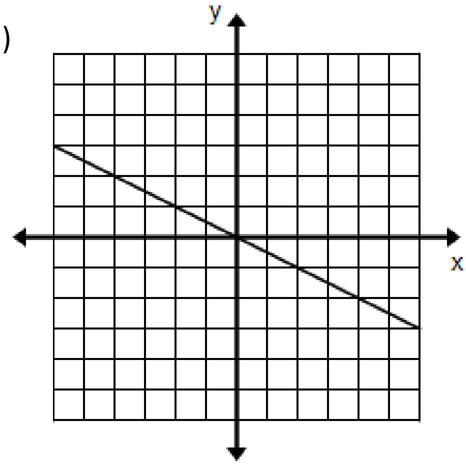

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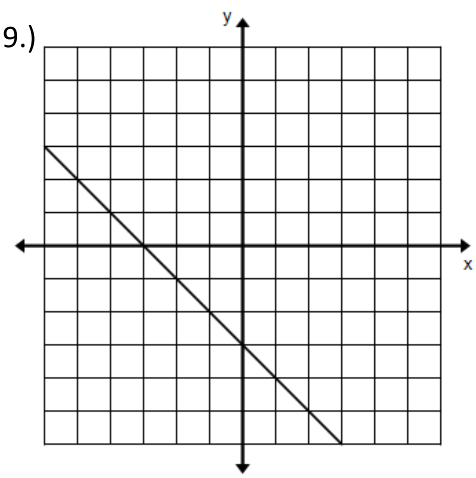

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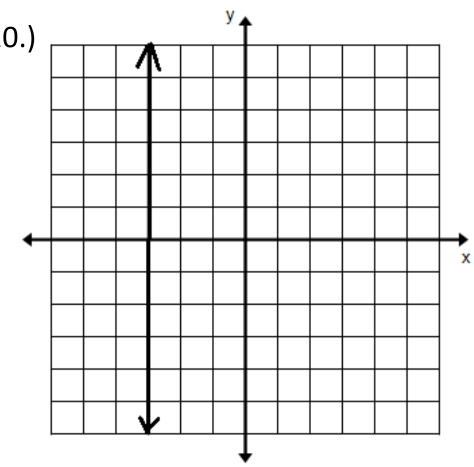

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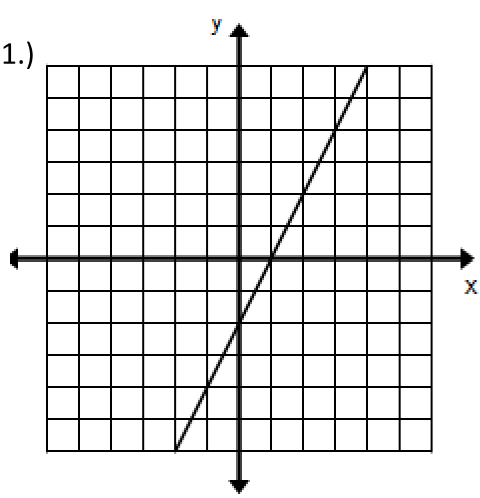

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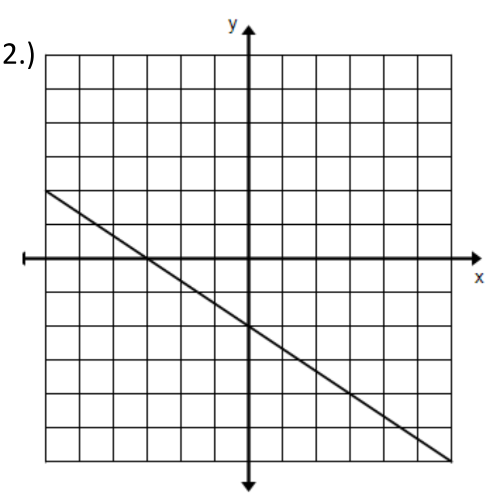


Determine the Slope of a Line Given Two Points

To find the slope of a line from two points, the **slope formula** must be used.

$$\textbf{Slope Formula: } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example

Find the slope of the line that goes through the points $(-3, 5)$ and $(2, -5)$.

First, label the points with x_1, x_2, y_1, y_2:

X_1 Y_1	X_2 Y_2
(-3, 5)	(2,-5)

Next, substitute the values into the slope formula; then simplify the fraction completely.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-5}{2-(-3)} = \frac{-10}{5} = -2$$

The slope of the line is -2.

Try These

Find the slope of the line that goes through the points listed. Show all work.

1. $(-4, -2)$ and $(3, 7)$
2. $(4, 3)$ and $(8, 0)$
3. $(2, -7)$ and $(-8, 3)$
4. $(9, -12)$ and $(-3, -6)$

5. $(-15, 0)$ and $(0, 11)$

6. $(-1, -11)$ and $(-3, -7)$

7. $(1, -5)$ and $(-6, -10)$

8. $(-20, 16)$ and $(4, -8)$

9. $(5, 2)$ and $(-4, -4)$

10. $(3, -3)$ and $(3, 3)$

11. $(-36, 13)$ and $(-8, -3)$

12. $(15, -4)$ and $(-15, -4)$

Determine the Slope of a Line Given an Equation

To find the slope of a line from an equation, the equation must be in slope-intercept form ($y = mx + b$). The m is the slope.

Therefore you must change the equation to slope-intercept form. To put an equation into slope-intercept form, solve the equation for y. The slope will be the number in front of the x (the coefficient of the x term).

Example 1

Find the slope of the equation: $\boldsymbol{y = \frac{3}{11}x + 2}$

This equation is already in $y = mx + b$ so we just need to find the m value → $m = \frac{3}{11}$

So the **slope =** $\boldsymbol{\frac{3}{11}}$

Example 2

Find the slope of the equation: $\boldsymbol{3x - 2y = 11}$

This equation is NOT in slope intercept form so we need to convert it to $y = mx + b$.

Move the x term to the other side: $-2y = -3x + 11$

Get the y all by itself by dividing by -2: $y = \frac{-3}{-2}x + \frac{11}{-2}$

Simplify the fractions: $y = \frac{3}{2}x - \frac{11}{2}$

This equation is now in slope-intercept form so we can get our slope: $m = \frac{3}{2}$

The slope of the line is $\boldsymbol{\frac{3}{2}}$

Try Some

Find the slope for each of these equations. Show all work.

1. $y = -\frac{7}{5}x + 2$
2. $2x - 8y = 1$

3. $y = 9x - 5$

4. $5x + y = 7$

5. $y = \frac{2}{3}x - 10$

6. $-2x - 7y = 8$

7. $y = -11x$

8. $x - 7y = 14$

9. $y = \frac{1}{8}x - 14$

10. $6x + 8y = 5$

11. $y = -7$

12. $-5x - 20y = 8$

13. $y = -2x + 4$

14. $8x + y = 4$

Converting a Linear Equation to Slope-Intercept Form
$y = mx + b$

The goal of converting an equation to Slope-Intercept Form is **to isolate y on one side of the equation**. Thus, to convert to slope-intercept form, perform inverse operations on terms until y stands alone on one side.

Example

Convert $4x + 6y = 7$ to slope-intercept form.	$4x + 6y = 7$
Subtract $4x$ from both sides	$-4x \qquad -4x$
	$6y = -4x + 7$
Divide both sides by 6	$\frac{6y}{6} = \frac{-4x+7}{6}$
	$y = \frac{-4}{6}x + \frac{7}{6}$
Simplify	$y = -\frac{2}{3}x + \frac{7}{6}$

So, $\mathbf{4x + 6y = 7}$ converts to $\mathbf{y = -\frac{2}{3}x + \frac{7}{6}}$

Try These

1.) $4x - 2y = -10$ 1. ______________________

2.) $x + 6y = 24$ 2. ______________________

3.) $3x - y = 8$ 3. ______________________

4.) $5x + 7y = 20$ 4. ______________________

5.) $2y - 3x = -42$

5. ______________________

6.) $5y + 2x = 15$

6. ______________________

7.) $6x = 14 - 2y$

7. ______________________

8.) $5x + 2 = \frac{1}{2}y + 6$

8. ______________________

9.) $7 - 2y = 6 - 2x$

9. ______________________

10.) $\frac{1}{3}y + 5 = 4 - x$

10. ______________________

Converting a Linear Equation to Standard Form
$Ax + By = C$

The goal of converting an equation to Standard or General Form is **to place x and y on one side of the equation and the constant term (the number) on the other side**. Then, if necessary, convert all coefficients to integers. If any of the coefficients or the constant are fractions, multiply the entire equation by the least common denominator of all the fractions. This will eliminate the fraction.

Example

Convert $y = \frac{2}{3}x - 5$ to Standard Form.	$y = \frac{2}{3}x - 5$
Flip the left and right sides	$\frac{2}{3}x - 5 = y$
Add 5 to both sides	$+5 \quad +5$
	$\frac{2}{3}x = y + 5$
Subtract y from both sides	$-y \quad -y$
	$\frac{2}{3}x - y = 5$
Eliminate the fractions by multiplying by 3	$3\left(\frac{2}{3}x - y\right) = 3(5)$
	$2x - 3y = 15$

So, $\boldsymbol{y = \frac{2}{3}x - 5}$ converts to $\boldsymbol{2x - 3y = 15}$

Try These

---

1. $y = -4x - 6$ 1. ______________________

2. $y = \frac{1}{4}x + 1$ 2. ______________________

3. $y = 8x - \frac{3}{2}$ 3. ______________________

4. $y = \frac{2}{5}x + \frac{1}{2}$

4. ______________________

5. $y = -3x + 10$

5. ______________________

6. $5y = \frac{1}{2}x - 2$

6. ______________________

7. $6x = 14 - 2y$

7. ______________________

8. $\frac{3}{5}x + 2 = \frac{1}{2}y + 6$

8. ______________________

9. $7 - 2y = 6 - 2x$

9. ______________________

10. $\frac{1}{3}y + 5 = 4 - x$

10. ______________________

Unit 3

Graphing Linear Equations

Objectives:

1. Identify, describe, and/or use constant rates of change
2. Graph an equation in slope-intercept form
3. Graph an equation in standard form
4. Graph horizontal and vertical lines

Objectives I must do for Test B: (circle ones you must do)

1 2 3 4

Identify, Describe and/or Use Constant Rates of Change

Constant rates of change are real-world examples of slope. Examples would be feet per second, days per week, ballots in an envelope, windows in a car, meals for a person, or people for a bus. Notice that there is the word "per", "for a", or "in a" between the two parts of the rate of change. Because slope is $\frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$, the first word represents the y-value and the second word represents the x-value. It is also important that the second word is for 1 unit.

Acceptable rates of change would be:

35 miles per hour $\frac{35\ miles}{1\ Hour}$	44 people for a bus $\frac{44\ people}{1\ bus}$	50 ballots in an envelope $\frac{50\ ballots}{1\ envelope}$

If there is a fraction, put the whole fraction in the numerator and put 1 in the denominator.

Such as: 7/2 cookies per person = $\frac{3\frac{1}{2}\ cookies}{1\ person}$ or 1/3 feet in a second = $\frac{\frac{1}{3}\ feet}{1\ second}$

Example

Sharon is a barber. After 2 days, she cut 15 people's hair. After 4 days, she cut 25 people's hair.

A. Identify the constant rate of change.

$(2, 15)$ and $(4, 25)$ are the two points given by the data. It is not $(15, 2)$ and $(25, 4)$. The reason for this is that one would note how many haircuts per day and NOT how many days per haircut.

$$\text{Use } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{25 - 15}{4 - 2} = 5$$

B. Describe the constant rate of change in haircuts per day.

There were 5 haircuts per day. It was not 5 days per haircut, as that doesn't make sense.

C. How many haircuts can be done in 7 days?

Since the rate of change is the slope, and therefore the value of m, an equation can be found for the problem.

$y = mx + b$ — Use $m = 5$ and either one of the points to find b. $m = 5$ and (2,15), so $x = 2$ and $y = 15$

$15 = 5 \cdot 2 + b$

$b = 5$

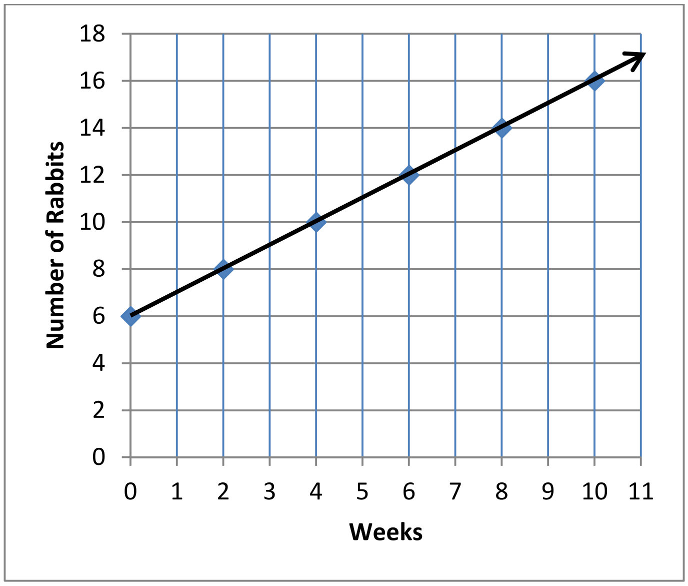
$y = 5x + 5$ — Since days are x-values, put 7 in place of x and find y, which will be the number of haircuts in 7 days.

$y = 5 \cdot 7 + 5$

$y = 40$ — 40 haircuts will be done in 7 days.

Example

The graph below shows the number of rabbits in a neighborhood over 10 weeks.



A. Identify the constant rate of change. Use $m = \frac{rise}{run} = \frac{2}{2} = 1$

B. Describe the constant rate of change in rabbits per week.

a. There was one more rabbit in the neighborhood per week.

C. How many rabbits would there be in 20 weeks?

a. Use $y = mx + b$. The slope is 1 and the graph intersects the y-axis at 6, so the equation is $y = x + 6$. The 20 weeks value is an x-value. Put 20 in for the x-value and solve for y. $y =$ 20 + 6 = 26 rabbits in 20 weeks.

Try These

Answer each question. Show all work used to find the answers.

1. Jayla was on a hiking trip. She hiked 41 miles in 2 days and 121 miles in 6 days.

A. Identify the constant rate of change.

B. Describe the constant rate of change in miles per day.

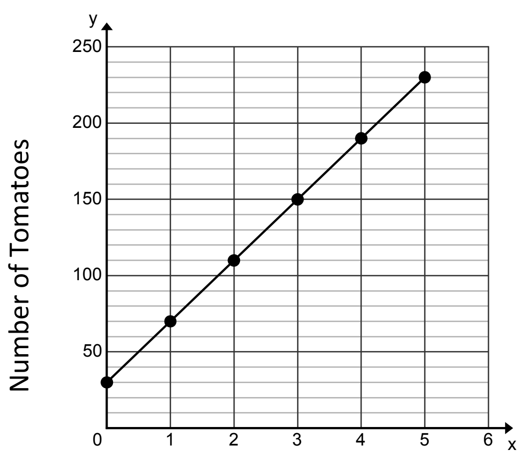
C. How many miles will she hike in 10 days?

2. Yasir was a realtor specializing in selling lots in a new housing development. He sold 17 lots in 2 weeks and 35 lots in 6 weeks.
   A. Identify the constant rate of change.

   B. Describe the constant rate of change in lots in a week.

   C. How many weeks will it take him to sell 53 lots?

3. Tyrone planted a tomato garden each year for 5 years. His results are in the graph below.



Years

   A. Identify the constant rate of change.

   B. Describe the constant rate of change in tomatoes per year.

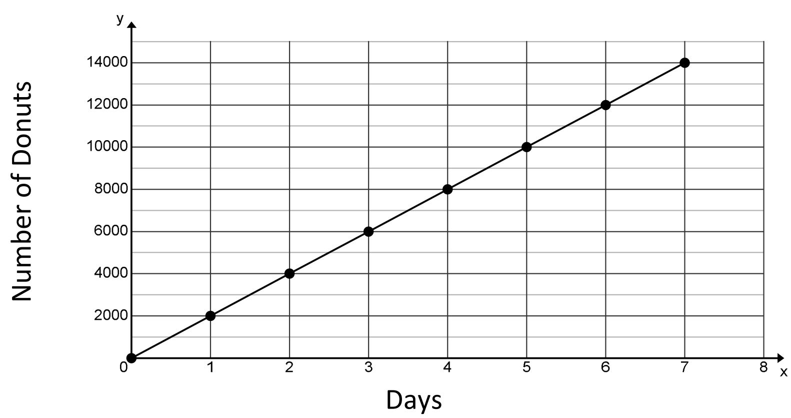
   C. How many tomatoes will Tyrone grow in 7 years?

4. Olga planted trees in a park. By the end of the third week, she had planted 56 trees. By the end of the fifth week, she had planted 70 trees.
    A. Identify the constant rate of change.

    B. Describe the constant rate of change in trees each week.

    C. How many trees can she plant in 8 weeks?

5. Mitzel's Bakery makes donuts over 7 days. The data is in the graph below.



    A. Identify the constant rate of change.

    B. Describe the constant rate of change in donuts in a day.

    C. How many days will it take to make 20150 donuts?

Graphing an Equation in Slope-Intercept Form

Slope-Intercept Form of an equation: $y = mx + b$

It is called slope-intercept form because m is the slope and b is the y-intercept.

We need to use both the slope and y-intercept to graph the equation so you need to identify these values before graphing.

When graphing:

a. Always plot the y-intercept first; once you plot this point you will not use the y-intercept again
b. From your point that you plotted, you use the slope to plot more points
c. Finally, you use a ruler to connect all of your points; remember to put arrows at the ends of the line since it continues in both directions

Example One

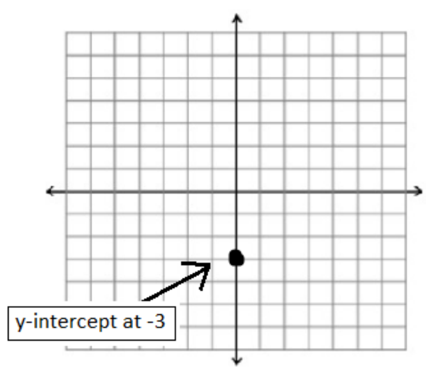
Graph the line $y = 2x - 3$

Step 1: Identify slope and y-intercept –

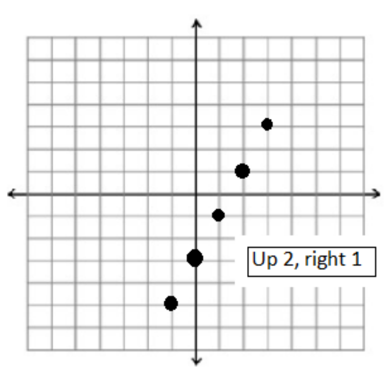
$$y = 2x - 3$$

Slope = 2 y-intercept = -3

Step 2: Graph the y-intercept (it will be on the y-axis)

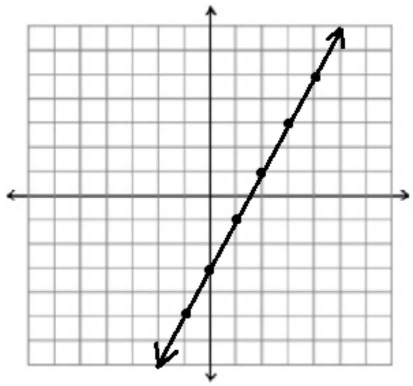


Step 3: Starting at the point you already plotted (the y-intercept), use the slope to plot more points. Since the slope is 2, we go up 2 right 1.



Remember: Slope $= \frac{rise}{run}$

Step 4: Use a ruler to connect your points. Put an arrow at each end since the line continues.



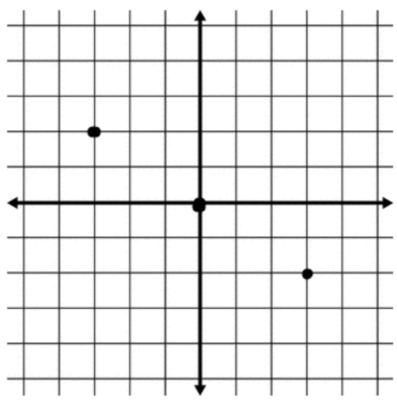
This graph is the final solution.

Example Two

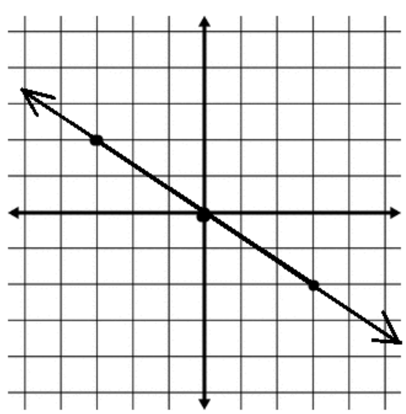
Graph the line $y = -\frac{2}{3}x$

Step 1: Slope = $-\frac{2}{3}$; y-intercept = 0 since there is no *b* value.

Steps 2 and 3: Graph the y-intercept and slope. Since the y-intercept is 0, we plot the intercept on the origin. From there use the slope. Since slope = $-\frac{2}{3}$, we go down 2, right 3.



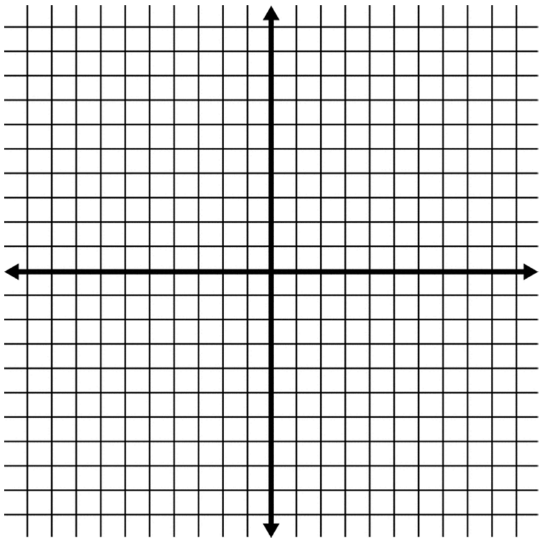
Step 4: Use your ruler to connect the points, and then put arrows at both ends.



Try Some! Graph each equation.

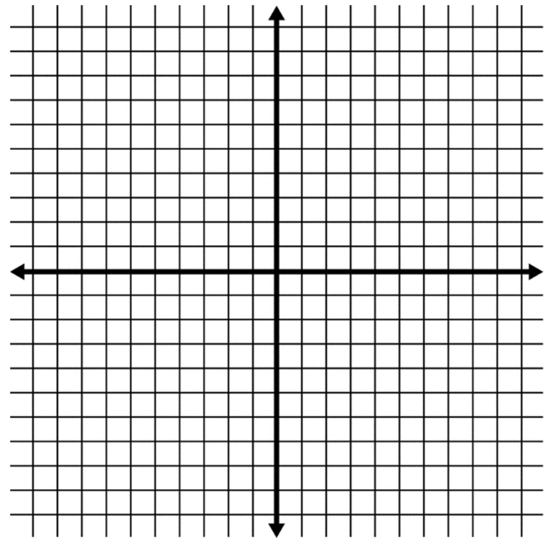
1.) $y = \frac{1}{2}x - 4$

y-int = slope =



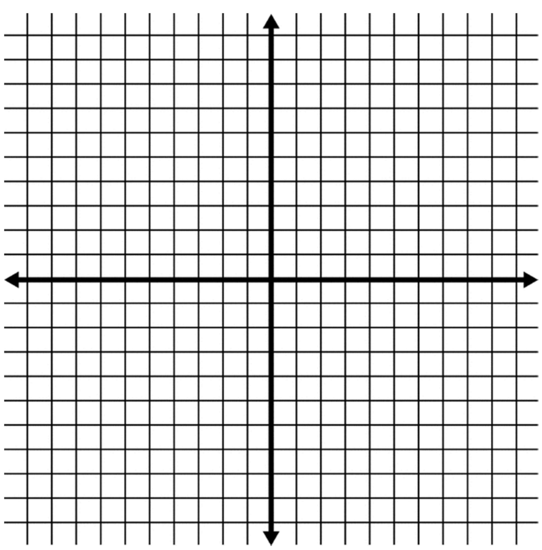
2.) $y = -x + 3$

y-int = slope =



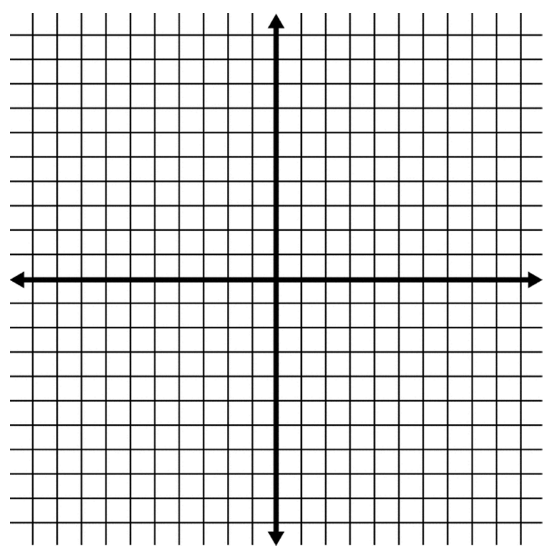
3.) $y = 3x - 1$

y-int = slope =



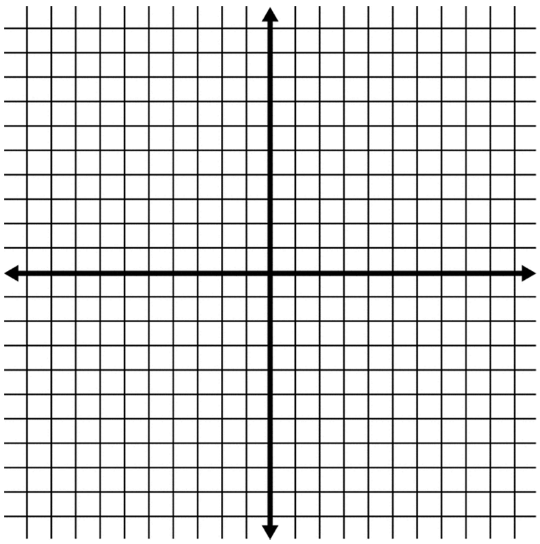
4.) $y = -\frac{3}{2}x + 1$

y-int = slope =



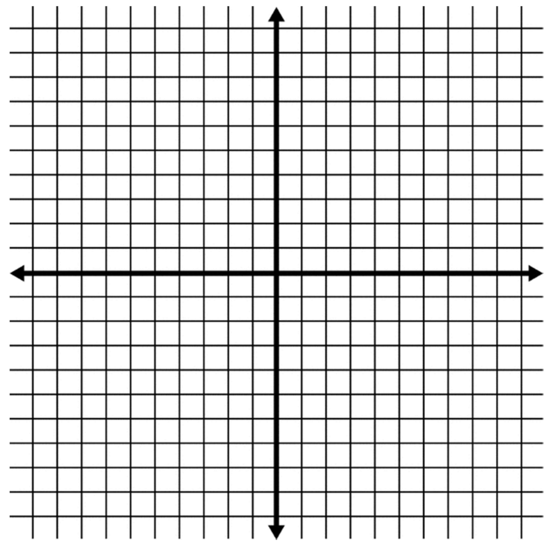
5.) $y = -\frac{1}{3}x$

y-int = slope =



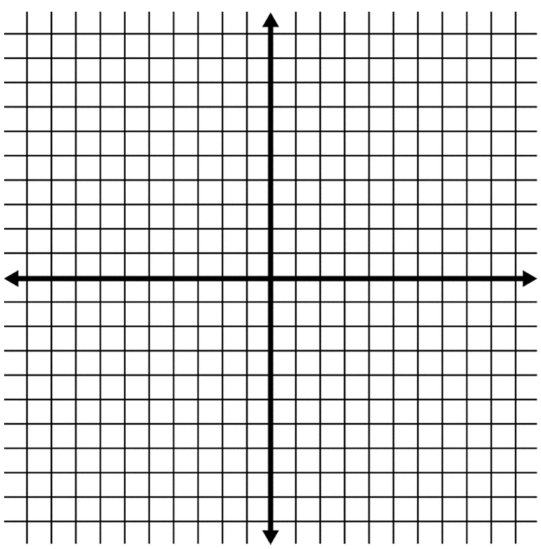
6.) $y = -2x + 4$

y-int = slope =



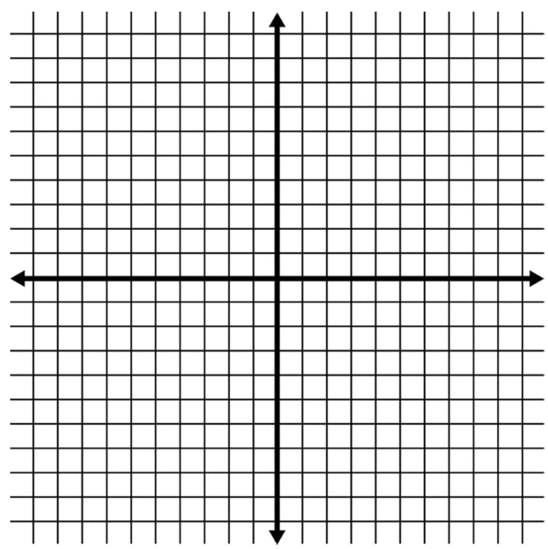
7.) $y = x - 4$

y-int = slope =



8.) $y = \frac{4}{3}x - 5$

y-int = slope =



Graphing an Equation in Standard Form

Standard form of a linear equation is $Ax + By = C$

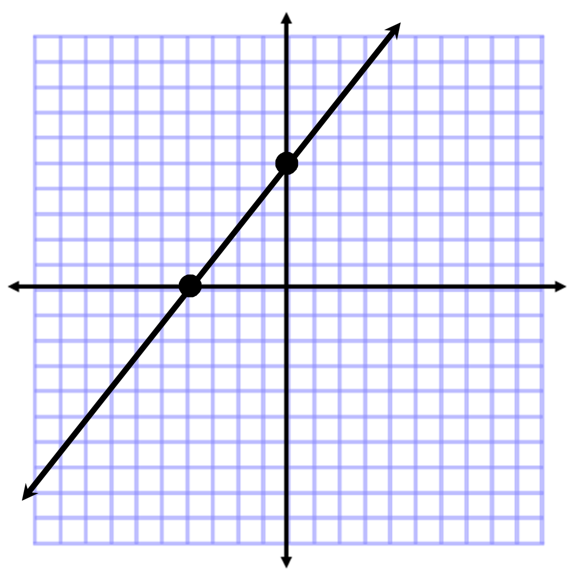
There are two methods to doing problems like this.

Method 1

For the first method, you must find the x-intercept and the y-intercept. Plot these points and then connect the points to graph the line.

Graph: $5x - 4y = -20$

To find the x-intercept, substitute 0 in for y and solve for x.	To find the y-intercept, substitute 0 in for x and solve for y.
$5x - 4(0) = -20$ $5x = -20$ $\frac{5x}{5} = \frac{-20}{5}$ $x = -4$	$5(0) - 4y = -20$ $-4y = -20$ $\frac{-4y}{-4} = \frac{-20}{-4}$ $y = 5$
x-intercept $(-4,\ 0)$	y-intercept $(0,\ 5)$



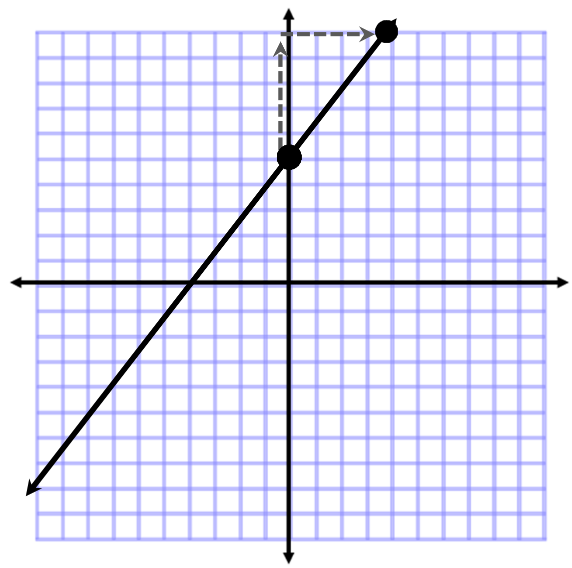
Method 2

The second way is to solve the equation for y and put the equation into Slope-Intercept form, as $y = mx + b$. Then graph the equation as we did in Objective 2.

<table>
<tr>
<td>Graph: $5x - 4y = -20$

Solve the equation for y.</td>
<td>$5x - 4y = -20$
$-5x \qquad\qquad -5x$
$-4y = -5x - 20$
$\frac{-4y}{-4} = \frac{-5x}{-4} + \frac{-20}{-4}$
$y = \frac{5}{4}x + 4$</td>
</tr>
<tr>
<td>The slope is $\frac{5}{4}$ and the y-intercept is $(0,4)$.

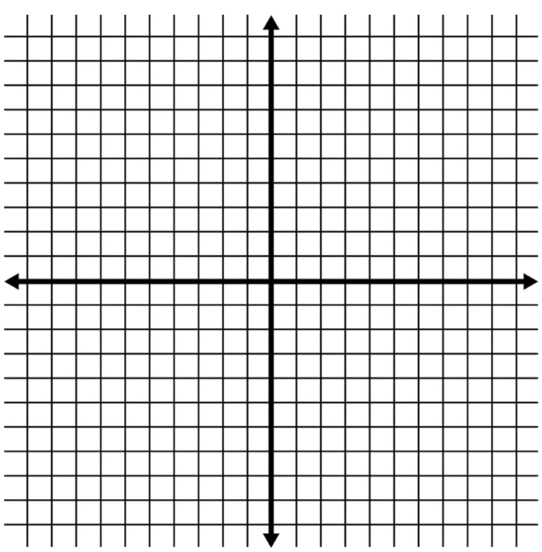
Start at the point $(0, 4)$ and rise 5 blocks from that point, then run 4 blocks to the right.

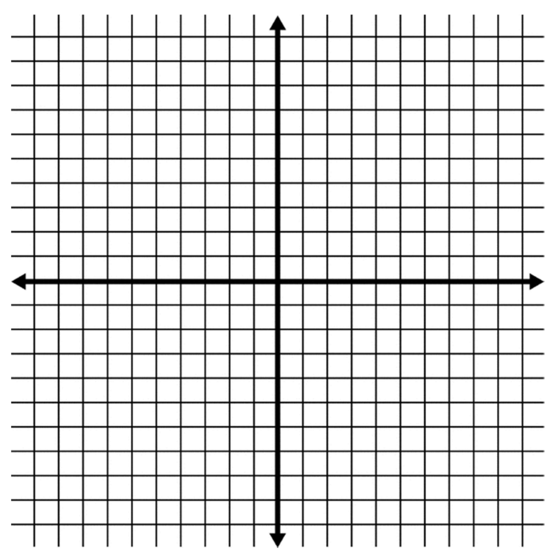
Connect the points.</td>
<td></td>
</tr>
</table>

Try These

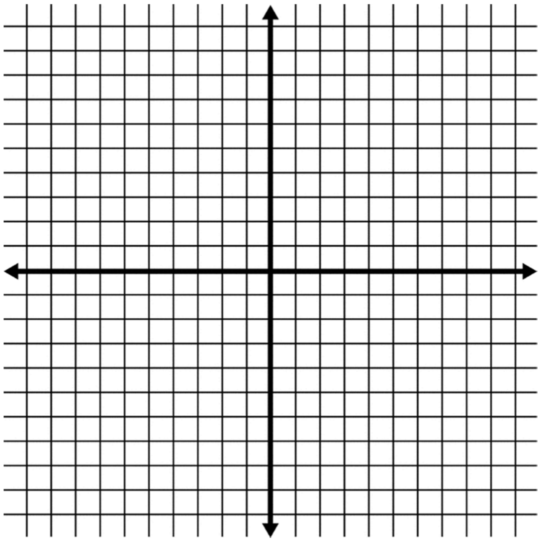
Graph each linear equation.

1. $3x + y = 6$
2. $5x - 2y = -10$

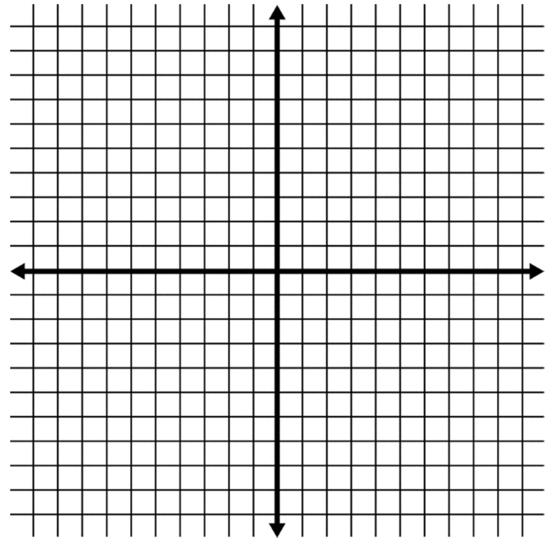




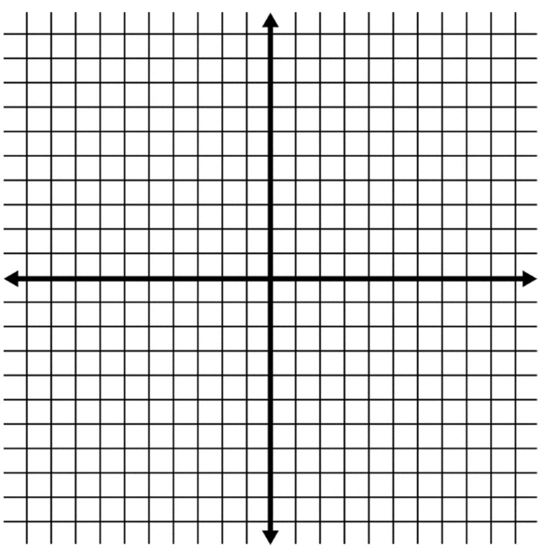
3. $2x - 8y = 4$



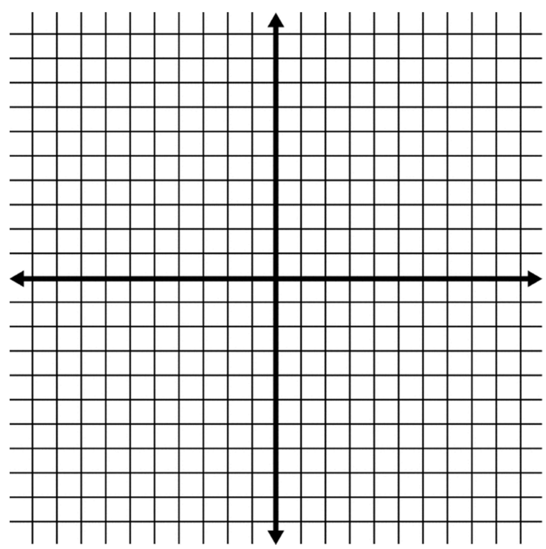
4. $9x + 6y = 12$



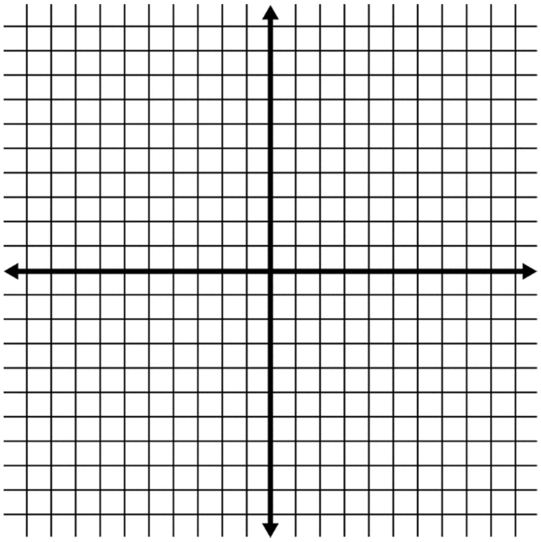
5. $x - 4y = -8$



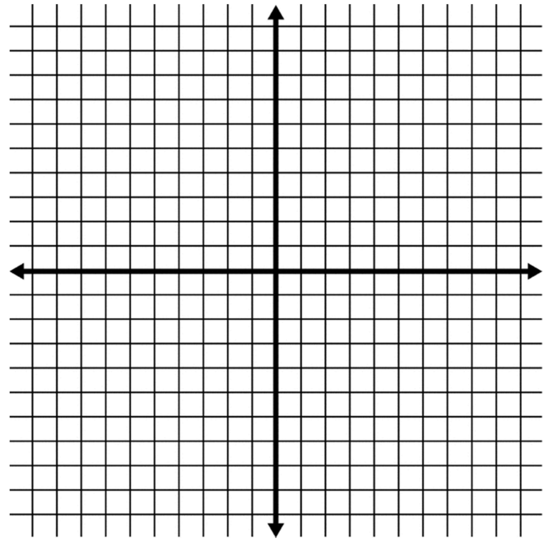
6. $x + y = 10$



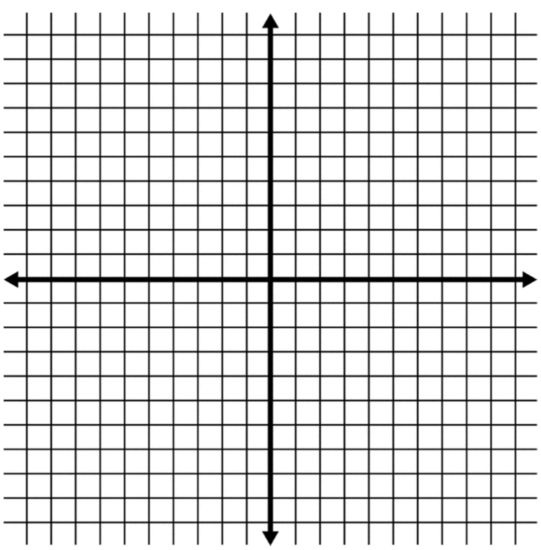
7. $7x - 2y = -10$



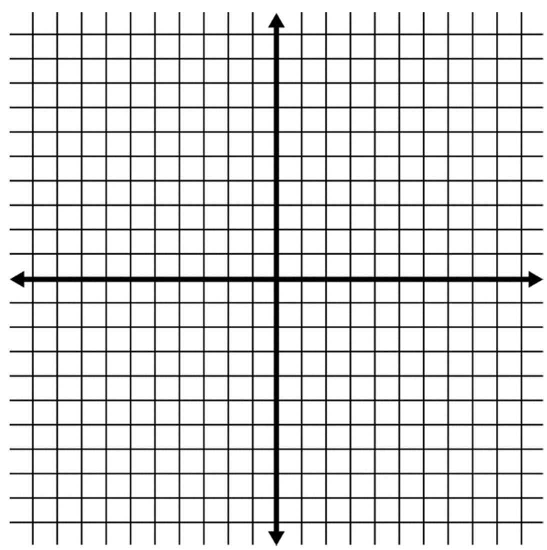
8. $2x + 3y = 9$



9. $6x + y = -6$



10. $4x - 5y = 20$



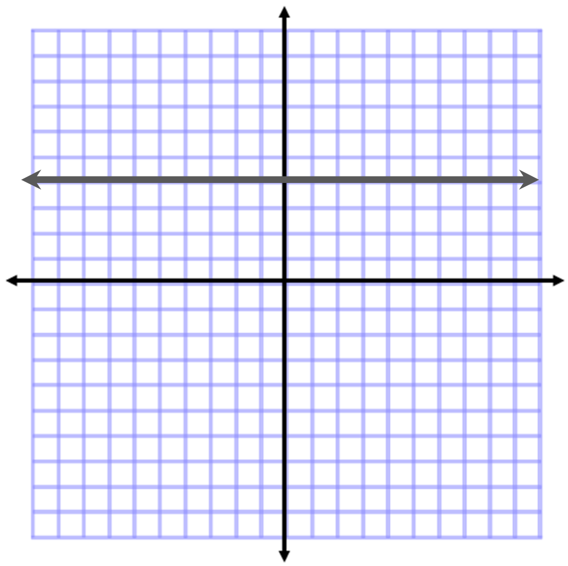
Graphing Horizontal and Vertical Lines

Horizontal lines will be written in the form $y = b$.

Since all y values will be the same for such a line, the x value may be any number. Look at the following example: $y = 4$.

x	y
-1	4
0	4
1	4
2	4

The y values must be 4. The x values may be any number. Plot all of these points and you will have a horizontal line.

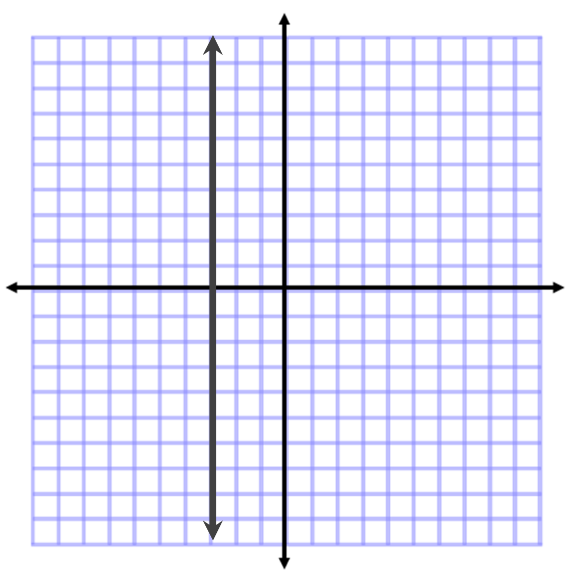


Vertical lines will be written in the form $x = a$.

Since all x values will be the same for such a line, the y values may be any number. Look at the following example: $x = -3$.

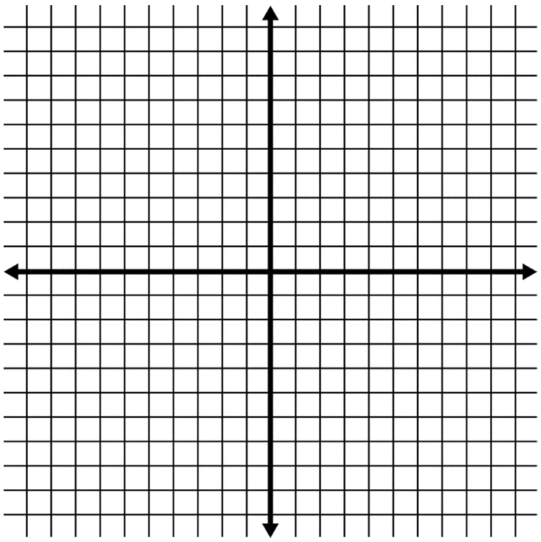
x	y
-3	-2
-3	-1
-3	0
-3	1

The x values must be -3. The y values may be any number. Plot all of these points and you will have a vertical line.

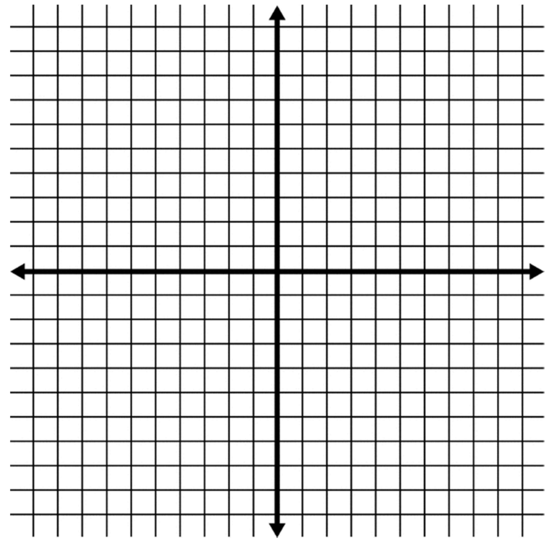


Try These: Graph each horizontal or vertical equation.

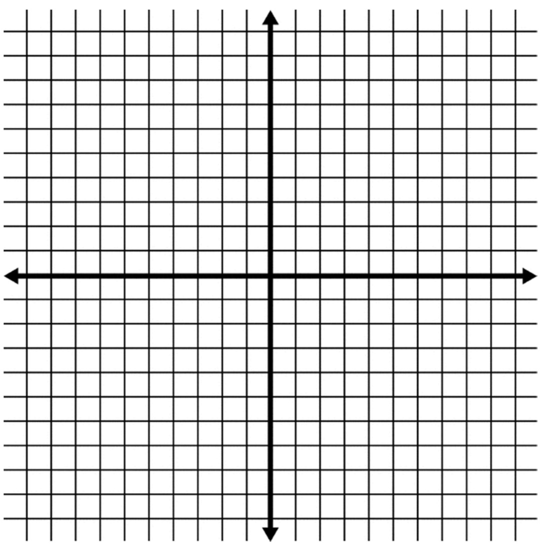
1. $y = 6$



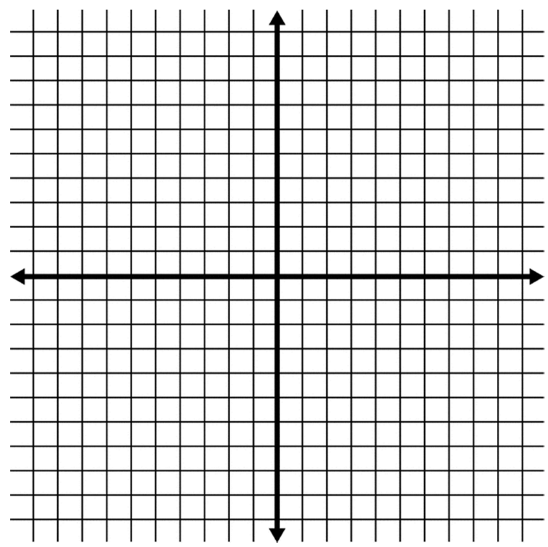
2. $x = -10$



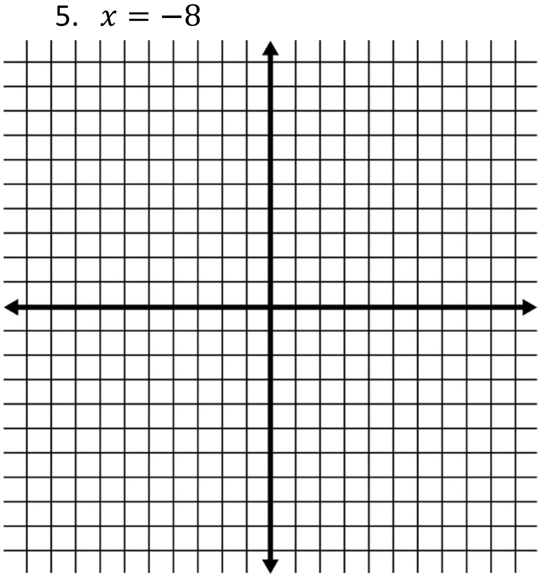
3. $x = 4$



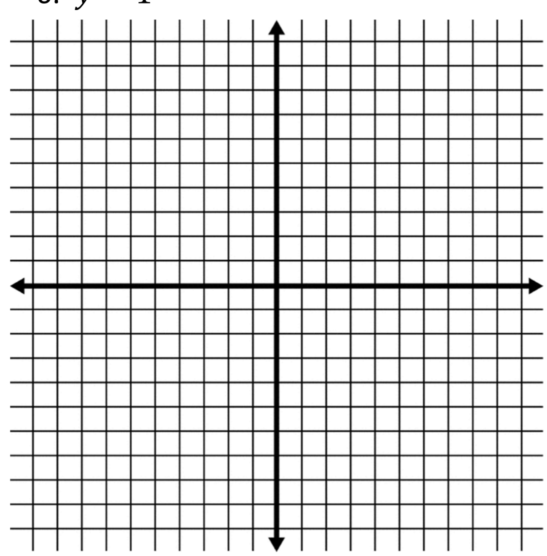
4. $y = -5$



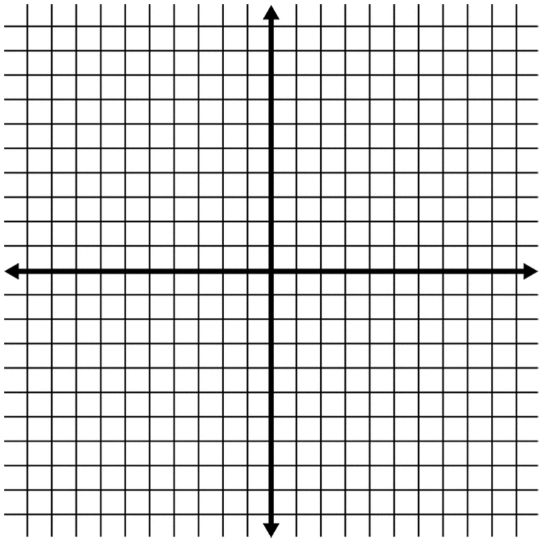
5. $x = -8$



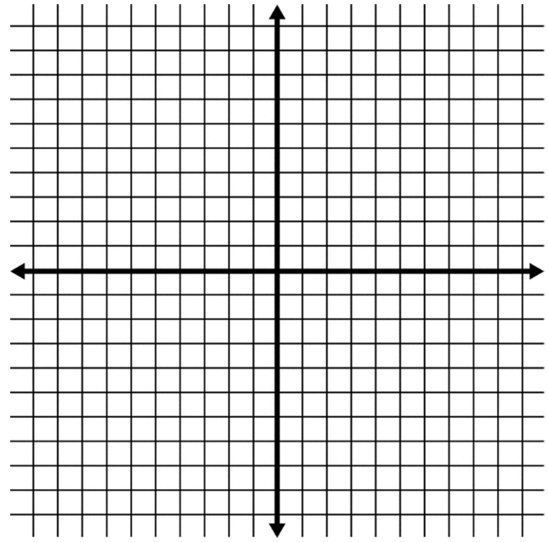
6. $y = 1$



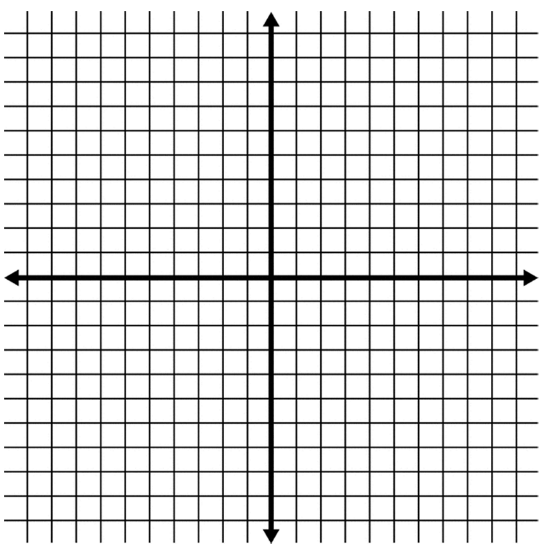
7. $y = -7$



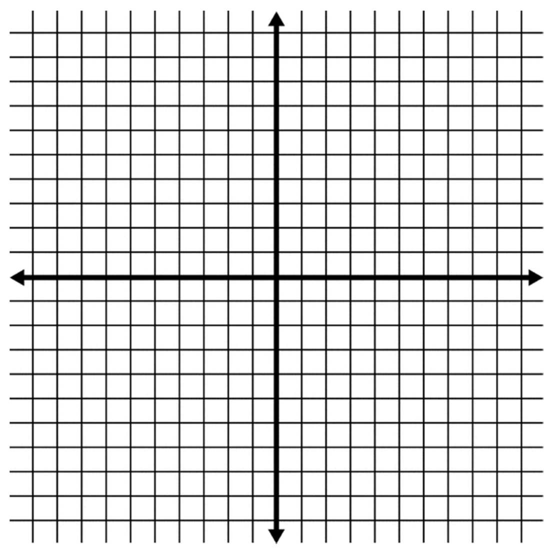
8. $x = 0$



9. $y = 0$



10. $x = 6$



Unit 4

Writing Linear Equations

Objectives:

1. Write or identify a linear equation given a graph
2. Write or identify the equation of horizontal and vertical lines given a graph
3. Write or identify a linear equation given the slope and point on the line
4. Write or identify a linear equation given two points on the line
5. Model a situation with a linear equation

Objectives I must do for Test B: (circle ones you must do)

1 2 3 4 5

Unit 4 Objective 1 Remediation

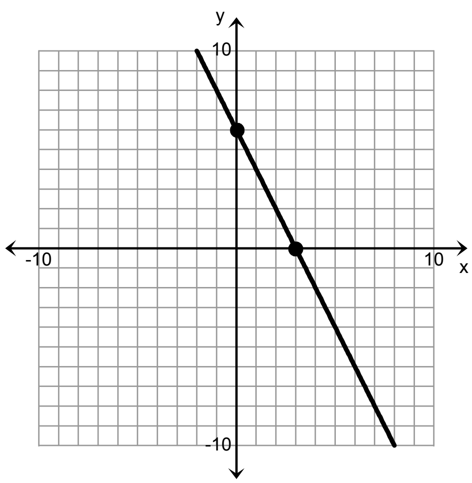
Writing the Equation of a Line from a Graph

Steps for Writing the Equation from the Graph

- **Find the slope, m, from the graph**
- **Find the y-intercept, b, from the graph**
- **Write the equation in slope-intercept form using the values you found for m and b**

$$y = \boxed{m}\,x + \boxed{b}$$

Example



Find two points on the line. From these two points, find the slope.

$$m = -\frac{6}{3} = -2$$

From the graph, find the y-intercept.

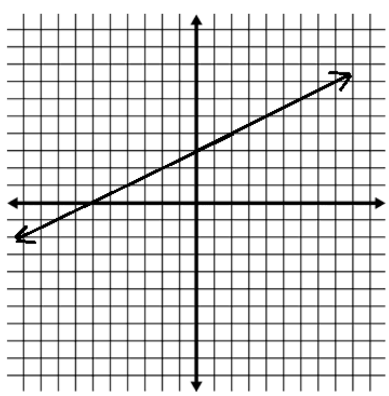
$$b = 6$$

Using slope-intercept form, put in the values for m and b.

Answer: $y = -2x + 6$

Try These

1.)

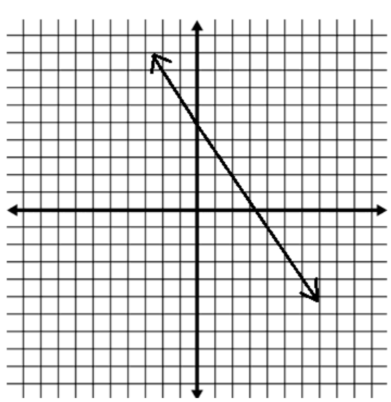


a.) What is the slope of the line?

b.) What is the y-intercept?

c.) Use your answers in a and b to write the equation of the line in slope-intercept form

2.)

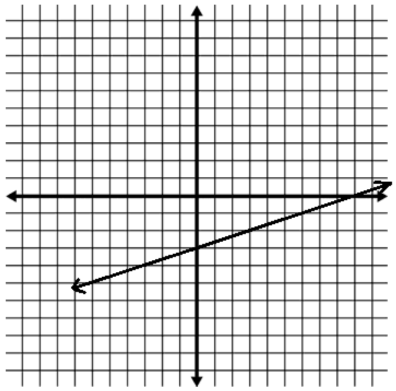


a.) What is the slope of the line?

b.) What is the y-intercept?

c.) Use your answers in a and b to write the equation of the line in slope-intercept form

3.)

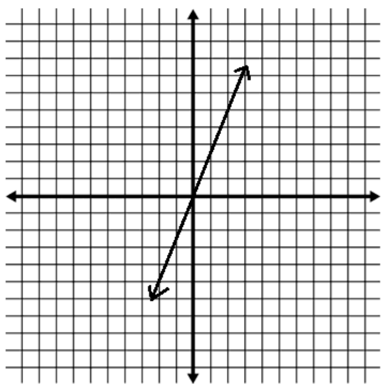


a.) What is the slope of the line?

b.) What is the y-intercept?

c.) Use your answers in a and b to write the equation of the line in slope-intercept form

4.)



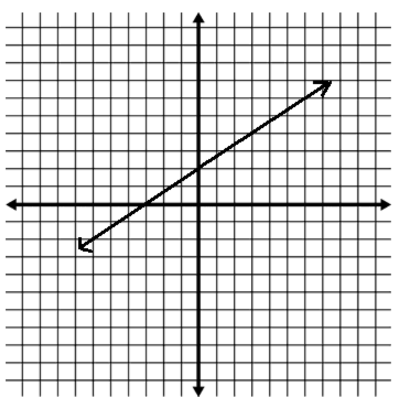
a.) What is the slope of the line?

b.) What is the y-intercept?

c.) Use your answers in a and b to write the equation of the line in slope-intercept form

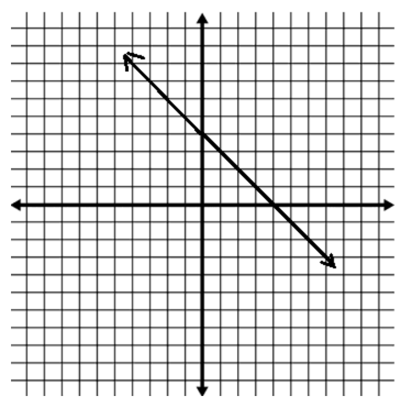
Write the equation, in slope-intercept form, of each line below.

5.)



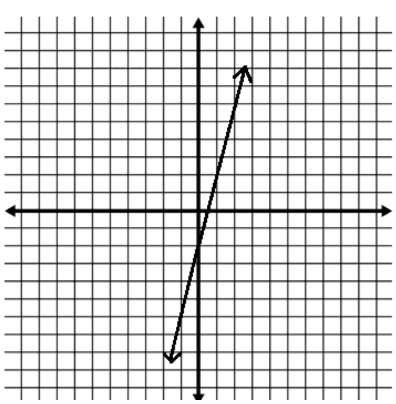
Equation: ______________________

6.)



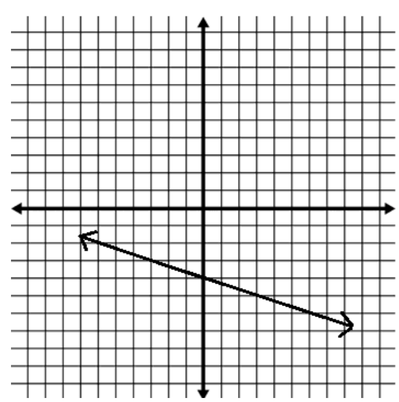
Equation: ____________________

7.)



Equation: ______________________

8.)



Equation: ______________________

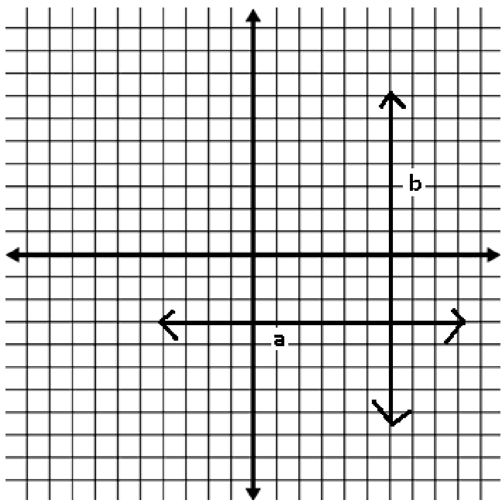
Unit 4 Objective 2 Remediation

Writing the Equation of a Horizontal or Vertical Line from a Graph

Equation of a Vertical Line: $x = a$

Equation of a Horizontal Line: $y = b$

Example:



Equation of Line a:
Line a is horizontal so it will be $y = b$

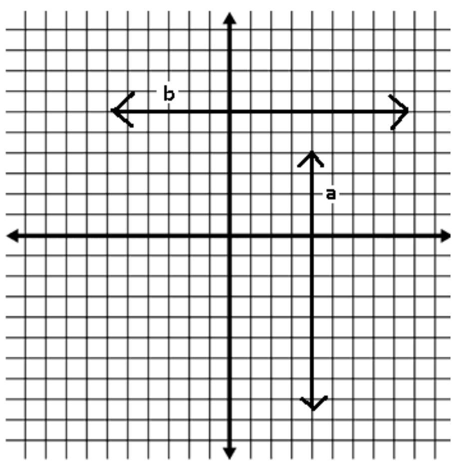
Since the line hits the y-axis at -3 the equation is: $\boldsymbol{y = -3}$

Equation of Line b:
Line b is vertical so it will be $x = a$

Since the line hits the x-axis at 6 the equation is: $\boldsymbol{x = 6}$

Write the equation for each line shown below.

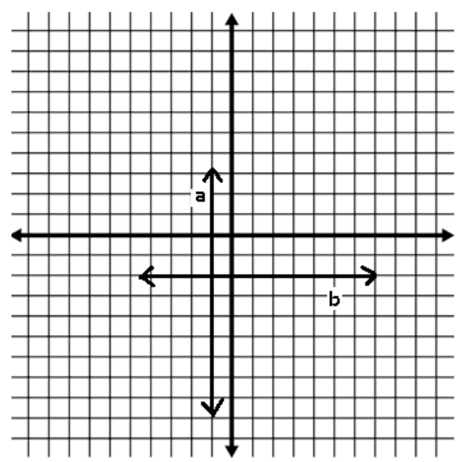
1.)



Equation of Line a: ______________

Equation of Line b: ______________

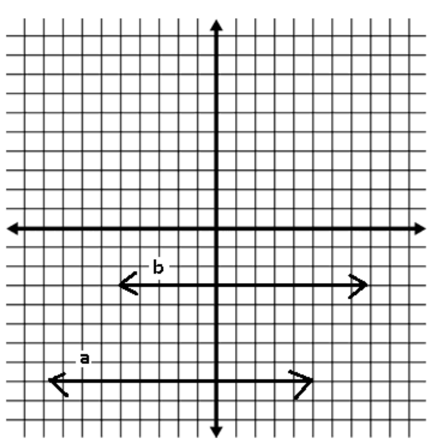
2.)



Equation of Line a: __________

Equation of Line b: __________

3.)



Equation of Line a: ____________

Equation of Line b: ____________

Write or Identify a Linear Equation Given the Slope and a Point

Slope-intercept Form: $y = mx + b$ where *m* is the slope of the line and *b* is the *y*-intercept.

- **Substitute the slope for *m***
- **Use the given point to substitute for *x* and *y***
- **Solve for *b***
- **Re-write the slope-intercept form substituting for *m* and *b***

Example One

Write the equation of the line in slope-intercept form.
Slope $= \frac{1}{2}$; Goes through (2, 3)

$$y = mx + b$$
$$3 = {}^{1}\!/_{2}\,(2) + b$$
$$3 = 1 + b$$
$$2 = b$$
$$\boxed{y = \frac{1}{2}x + 2}$$

Example Two

Write the equation of the line in slope-intercept form.
Slope $= -3$; Goes through $(-1, -5)$

$$y = mx + b$$
$$-5 = -3(-1) + b$$
$$-5 = 3 + b$$
$$-2 = b$$
$$\boxed{y = -3x - 2}$$

Try These

Write the equation in slope-intercept form.

1. Slope = 2 ; Line goes through the point $(2, 1)$

2. Slope = 3 ; Line goes through the point $(4, -2)$

3. Slope = $\frac{3}{4}$; Line goes through the point $(-8, 0)$

4. Slope = -1 ; Line goes through the point $(5, -2)$

5. Slope = 0 ; Line goes through the point $(3, -6)$

6. Slope = $-\frac{1}{2}$; Line goes through the point $(2, 5)$

7. Slope = $\frac{2}{3}$; Line goes through the point $(-1, 3)$

8. Slope = -3 ; Line goes through the point $(5, -1)$

9. Slope = -6 ; Line goes through the point $(2, 0)$

10. Slope = $\frac{1}{2}$; Line goes through the point $(5, -3)$

Write or Identify a Linear Equation Given Two Points

Slope-intercept Form: $y = mx + b$ where *m* is the slope of the line and *b* is the y-intercept.

- **Find the slope of the line $m = \frac{y_2 - y_1}{x_2 - x_1}$**
- **Use the one of the given points to substitute for *x* and *y***
- **Substitute in the value you found for *m***
- **Solve for *b***
- **Re-write the slope-intercept form substituting for *m* and *b***

Example One

Write the equation of the line in slope-intercept form that goes through the points (0, 2) and (2, 3)

$$m = \frac{3-2}{2-0} = {}^{1}\!/_{2}$$
$$y = mx + b$$
$$3 = {}^{1}\!/_{2}\,(2) + b$$
$$3 = 1 + b$$
$$2 = b$$
$$\boxed{y = \frac{1}{2}x + 2}$$

Example Two

Write the equation of the line that goes through the points $(-5, 3)$ and $(1, -3)$

$$m = \frac{-3-3}{1-(-5)} = \frac{-6}{6} = -1$$
$$y = mx + b$$
$$-3 = -1(1) + b$$
$$-3 = -1 + b$$
$$-2 = b$$
$$\boxed{y = -x - 2}$$

Try These

Write the equation in slope-intercept form.

1. $(3, 4)$ and $(2, 1)$

2. $(-2, 4)$ and $(4, -2)$

3. $(-1, 4)$ and $(-8, 0)$

4. $(4, 5)$ and $(5, -2)$

5. $(3, -4)$ and $(3, -6)$

6. $(-2, -5)$ and $(2, 5)$

7. $(3, 2)$ and $(-1, 3)$

8. $(4, 3)$ and $(5, -1)$

Unit 4 Objective 5 Remediation

Writing and Applying Linear Models

Example One

Membership to the local gym costs $99 plus a monthly fee of $25.

A. Write an equation that represents cost C of being a member for m months.

Since it cost $25 each month, this is the slope. $m = 25.$
The one-time fee of $99 is the y-intercept. $b = 99$
Using these two values, write the equation. $C = 25m + 99$

B. How much would it cost to be a member for 24 months?
Substitute 24 in for m and solve for C.

$$C = 25(24) + 99$$
$$C = \$699$$

It costs $699 for 24 months at the local gym.

C. If it cost $249, for how many months would you be a member?
Substitute $249 for C and solve for m.

$$249 = 25m + 99$$
$$150 = 25m$$
$$m = 6$$

6 months cost $249 at the local gym.

Try These

1. Jillian is opening a new savings account with a gift of $200 from her grandmother. She plans of putting $25 each week into the account.
   a. Write an equation for the total savings S of putting money in the savings account for w weeks.

   b. How much would she have after 8 weeks?

2. Steven wants to have a stand at a fair to sell personalized license plates. It costs $100 per day after paying a setup fee of $500.
   a. Write an equation for the total cost C of having the stand for d days.

   b. If it cost Steven $1,300, how many days was he at the fair?

3. You want to have your lawn treated by a local lawn company. It will cost $150 to have your lawn evaluated and $300 per quarter of a year to have treatments given to your lawn.
   a. Write an equation for the total cost C to have your lawn treated q quarters of a year.

   b. How much would it cost for 2 years?

Example Two

Bruce is having electrical problems and calls an electrician. The electrician charges a service fee plus $32 per hour he works. For two hours of work he charges $103 and for five hours of work he charges $199.

Write an equation that represents cost C of hiring the electrician for x hours of work.

Since it cost $32 per hour, the slope is 32.	$m = 32$
From the problem, we write two points.	$(2, 103),\ (5,\ 199)$

We must find the y-intercept using the slope $m = 32$ and one of the points $(2,\ 103)$.

$$y = mx + b$$
$$103 = 32(2) + b$$
$$103 = 64 + b$$
$$b = 39$$

Using these two values, write the equation. $C = 32x + 39$

Try These

---

4. To order boxes of paper for a company, the options are 10 boxes for $150 and 15 boxes for $200.
   a. Write an equation for the cost C of b number of boxes of paper.

   b. How much would it cost for 75 boxes of paper?

5. At a bridal store, 2 bridesmaid dresses cost $900 and 5 bridesmaid dresses cost $1800.
   a. Write an equation for the C cost of d dresses.

   b. How many bridesmaid dresses could be bought with $2400?

6. When Paula orders books online for her book club, she must pay a shipping fee in addition to the cost of each book. She ordered 35 books and it cost her \$152. On her next order, it cost her \$140 for 32 books.
    a. Write an equation for the total cost C of b number of books.

    b. How much would it cost if she ordered 10 books?

    c. Paula received a bill that stated she owed \$116. How many books did she order?

7. Jillian has her own business selling pastries. When a customer uses a credit card, she must pay the credit card company a flat fee plus a fee per transaction. In the month of May she had 54 credit card transactions that cost her \$89. In the month of June she had 67 credit card transactions that cost her \$102.
    a. Write an equation for Jillian's cost C for n number of credit card transactions.

    b. In the month of July Jillian had 42 credit card transactions. What did the credit card company charge her?

    c. In the month of August Jillian received a bill from the credit card company stating that she owed \$119. How many credit card transactions did she have in August?

8. Kyle buys new car for \$25,900. Every month the car decreases in value. After 6 months the car is worth \$24,850. After 1 year (12 months) the car is worth \$23,800.
    a. Write an equation that represents the value of Kyle's car V after m months.

    b. After how many months would the car be worth nothing?

    c. How much is the car worth 5 years later? (Remember m = months)

Unit 5

Inequalities

Objectives:

1. Identify inequality symbols
2. Graph an inequality on a number line
3. Graph a compound inequality on a number line
4. Solve linear inequalities
5. Solve compound linear inequalities
6. Solve absolute value compound inequalities
7. Model situations using linear inequalities

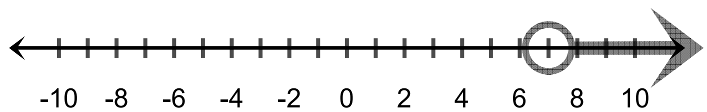
Objectives I must do for Test B: (circle ones you must do)

1 2 3 4 5 6 7

Graphing Inequalities on a Number Line

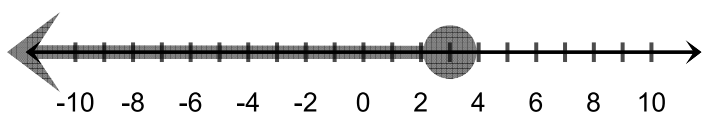
Example One: Graph: $x > 7$

First, we need to mark our number line.
Since there is no line under the symbol, x cannot equal 7, so we put an open dot above 7.
In this case, x is bigger than 7, so we draw the arrow to the right.



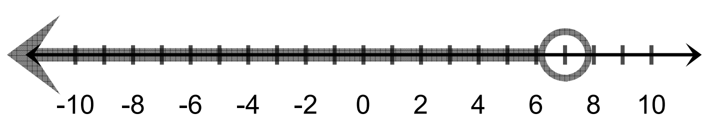
Example Two: Graph: $x \leq 3$

First, we need to mark our number line.
Since there is a line under the symbol, x can equal 3, so we put a solid dot above 3.
In this case, x is smaller than 3, so we draw the arrow to the left.



Example Three: Graph: $7 > x$

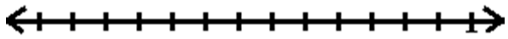
First, **re-write as $x < 7$**.
Next, we need to mark our number line.
Since there is no line under the symbol, x cannot equal 7, so we put an open dot above 7.
In this case, x is smaller than 7, so we draw the arrow to the left.



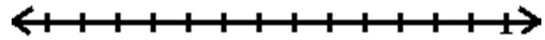
If the variable is on the right,
change it to the left BEFORE you graph.

Graph each inequality on the number line provided.

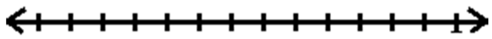
1) $-3 \leq x$



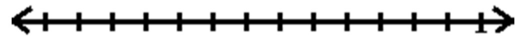
2) $x \geq 3$



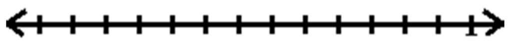
3) $-5 \leq x$



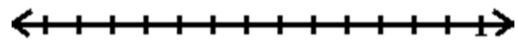
4) $2 > x$



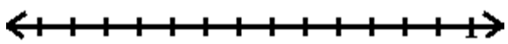
5) $x < -3$



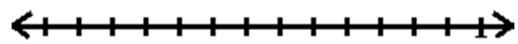
6) $-1 < x$



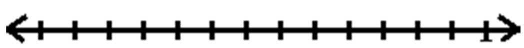
7) $x \geq -2$



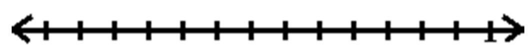
8) $x > -1$



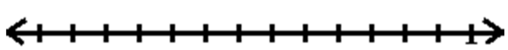
9) $x \leq -4$



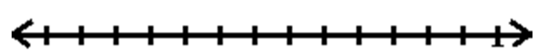
10) $3 > x$



11) $x \geq 7$



12) $-3 < x$



Graphing Compound Inequalities on a Number Line

Review on graphing inequalities:

1. First, identify the compound inequality as a Conjunction (And) or a Disjunction (Or).
2. If the compound inequality is a Conjunction (AND) you be shading between the two numbers.
3. If the compound inequality is a Disjunction (OR) you will be shading in opposite directions.

Example One

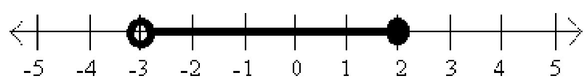
Graph: $-3 < x \leq 2$

This is a Conjunction since x is between the two numbers.

Put an open dot on -3 since it is a $<$ symbol – this means that x is greater than -3

Put a closed dot on 2 since it is a $\leq$ symbol – this means that x is less than or equal to 2

Shade between the two numbers.



Example Two

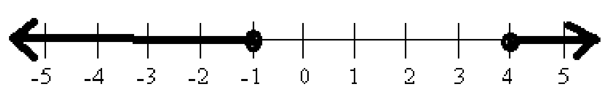
Graph: $x \leq -1$ **OR** $x \geq 4$

This is a Disjunction since there is an OR

Put a closed dot on -1 since it is a $\leq$ symbol – this means x is less than or equal to -1

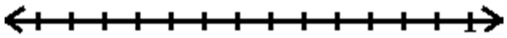
Put a closed dot on 4 since it is a $\geq$ symbol – this means x is greater than or equal to 4

Shade the tails since this is a disjunction.

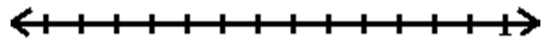


Graph each compound inequality on the number line provided.

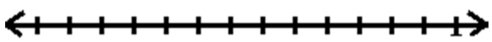
1. $-3 \le x < 1$



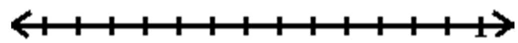
2. $x \ge 3$ OR $x \le -1$



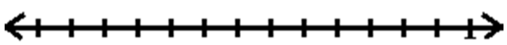
3. $-3 \le x \le 0$



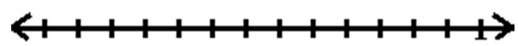
4. $2 > x > -3$



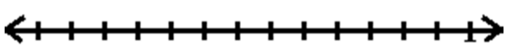
5. $x < -3$ OR $x \ge 2$



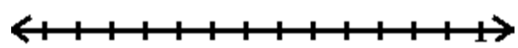
6. $-1 < x$ OR $x < -3$



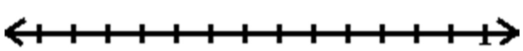
7. $x \ge 0$ OR $x \le -3$



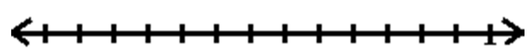
8. $-1 < x \le 3$



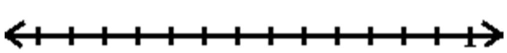
9. $-6 \le x \le -4$



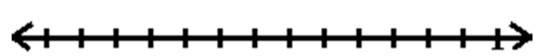
10. $x > 3$ OR $x < 1$



11. $x \ge 5$ OR $x < 3$



12. $-3 < x \le 2$



Solving Linear Inequalities

Remember that you switch the inequality sign if you are…

- Multiplying by a negative number
- Dividing by a negative number
- Switching the left and right sides of an inequality

Example One

Solve $30 \geq 2x + 10$

$$30 \geq 2x + 10$$

Subtract 10 from both sides

$$-10 \qquad -10$$

$$20 \geq 2x$$

Divide both sides by 2

$$\frac{20}{2} \geq \frac{2x}{2}$$

$$10 \geq x$$

Switch the left and right sides

$$\boldsymbol{x \leq 10}$$

Example Two

Solve $-2(2x - 3) < 5$

$$-2(2x - 3) < 5$$

Distribute -2

$$-2x + 6 < 5$$

Subtract 6 from both sides

$$-6 \quad -6$$

$$-2x < -1$$

Divide both sides by -2

$$\frac{-2x}{-2} > \frac{-1}{-2}$$

Switch the inequality sign!

$$\boldsymbol{x > \frac{1}{2}}$$

Example Three

Solve $2x - 13 \leq -3x + 2$

$$2x - 13 \leq -3x + 2$$

Add $3x$ to both sides

$$+3x \qquad +3x$$

$$5x - 13 \leq 2$$

Add 13 to both sides

$$+13 \quad +13$$

$$5x \leq 15$$

Divide both sides by 5

$$\frac{5x}{5} \leq \frac{15}{5}$$

$$\boldsymbol{x \leq 3}$$

Solve the following inequalities.

1.) $25 < 2x + 5$

2.) $-x - 7 \geq -9$

3.) $14x + 2 < -26$

4.) $-4 + \frac{1}{2}x \geq -12$

5.) $10 > 2(x - 1)$

6.) $-(2x - 6) \geq -14$

7.) $\frac{1}{6}(12x + 6) > 7$

8.) $4 \leq -\frac{3}{8}(16x - 8)$

9.) $5 - x > 9 + x$

10.) $-4 + 5x \geq 3 - x$

11.) $4x + 1 < 2x + 5$

12.) $\frac{1}{2}x \leq 2 + x$

Unit 5 Objective 5 Remediation

Solve Compound Linear Inequalities

Compound inequality – Two inequalities that are combined into one statement by the word and or or.

The graph of an AND compound inequality is the ***intersection*** *or overlapping* region of the two parts of the inequality. This type of compound inequality is called a ***conjunction****.*

The other type of compound inequality using OR is called ***disjunction.*** A compound inequality involving OR has solution regions that are the **union** or the total of the solution regions of the separate parts of the inequality.

Example One

Solve: $\mathbf{-5 \leq 3x + 7 \leq 10}$

Solve each inequality separately and then combine the results to form a compound inequality.

$-5 \leq 3x + 7$	AND	$3x + 7 \leq 10$
$-12 \leq 3x$		$3x \leq 3$
$-4 \leq x$		$x \leq 1$

The solution is $\mathbf{-4 \leq x \leq 1}$

Example Two

Solve: $\mathbf{2x + 1 \leq 13 \quad OR \quad x - 5 \geq -5}$

$2x + 1 \leq 13$	OR	$x - 5 \geq -5$
$2x \leq 12$		$x \geq 0$
$x \leq 6$		

The solution is $\mathbf{x \leq 6 \quad OR \quad x \geq 0}$

Try These

Solve the inequalities.

1. $2x > 8$ or $-3x \leq 15$
2. $2x + 1 \geq 5$ and $x - 8 < 2$

3. $1 + 2x > 5$ and $8 - x < 7$

4. $-1 < 2x - 1 < 11$

5. $2(x - 2) < 4$ or $4 \leq 2(x - 2)$

6. $-14 > 5x + 6 > -4$

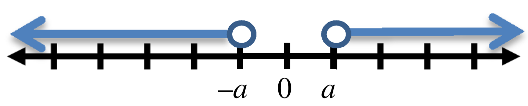
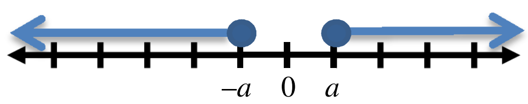
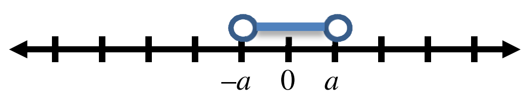
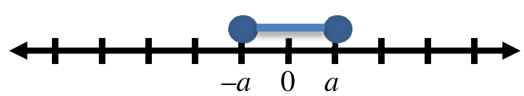
7. $2x - 4 > -2$ or $2x - 6 < -2$

8. $-3 \leq 2x + 15 < 21$

9. $2x + 1 < 1$ or $x + 5 > 8$

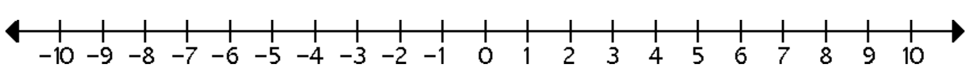
10. $2x - 5 < -5$ or $3x - 2 > 1$

Solve Absolute Value Compound Inequalities

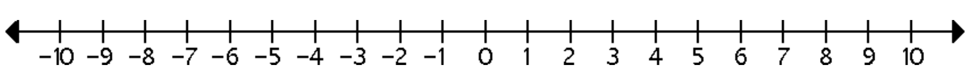
	What does it mean?	What does it look like?	How do we write it?
$\lvert x\rvert > a$	x is all the numbers more than a units away from 0 on the number line		$x < -a$ or $x > a$ disjunction
$\lvert x\rvert \geq a$	x is all the numbers more than or equal to a units away from 0 on the number line		$x \leq -a$ or $x \geq a$ disjunction
$\lvert x\rvert < a$	x is all the numbers less than a units away from 0 on the number line		$-a < x < a$ conjunction
$\lvert x\rvert \leq a$	x is all the numbers less than or equal to a units away from 0 on the number line		$-a \leq x \leq a$ conjunction

Graph the following.

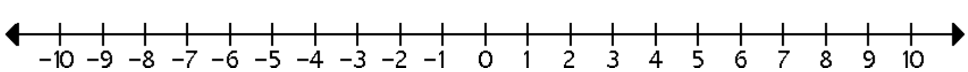
1. $|x| > 3$



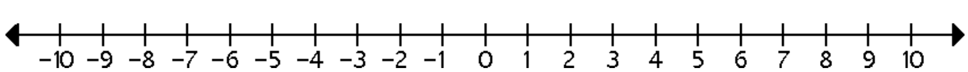
2. $4 \geq |x|$ **Hint:** Put the absolute value on the left **FIRST**! $\Rightarrow$ $|x| \leq 4$



3. $|x| < 4$



4. $|x| \geq 5$



Example One – Disjunction

Solve $|2x-4|>6$

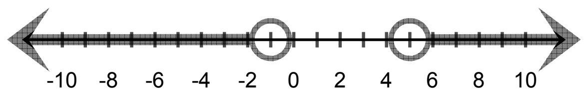
Rewrite the problem as a disjunction and solve.

$2x-4<-6 \quad \text{or} \quad 2x-4>6$

$2x<-2 \qquad\qquad 2x>10$

$x<-1 \quad \text{or} \quad x>5$

Graph the solution



Example Two – Conjunction

Solve $|5-3x|-3\leq 1$

Isolate the absolute value before you change it to a compound inequality

$|5-3x|\leq 4$

Rewrite the problem as a conjunction and solve.

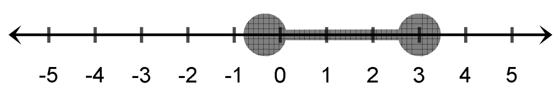
$-4\leq 5-3x\leq 4$ Subtract 5

$-9\leq -3x\leq -1$ Divide by -3, so don't forget to switch signs

$3\geq x\geq -\frac{1}{3}$

$-\frac{1}{3}\leq x\leq 3$

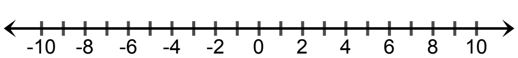
Graph the solution

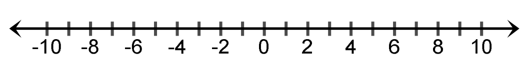


Try These

1. $|x+1|>3$

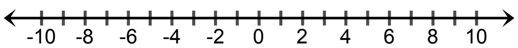
2. $|-9x|\leq 54$

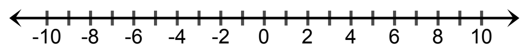




3. $|-2x - 6| \geq 5$

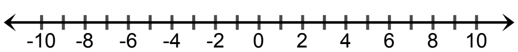
4. $|6 + 9x| + 4 \leq 28$

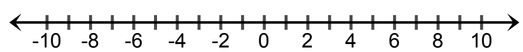




5. $|-8x - 3| > 11$

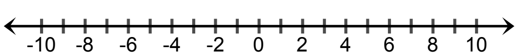
6. $|4x + 10| - 6 \geq 8$

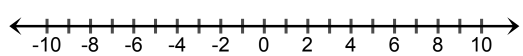




7. $-5 > |x + 5| - 6$

8. $9|x + 8| + 10 < 55$





Modeling Inequalities

$>$	$\geq$	$<$	$\leq$
is more than is greater than is larger than above	minimum at least is not less than no less than not smaller than	is smaller than is less than is fewer than below	maximum at most is not more than no more than is not greater than

Examples

1. Grace's grandfather is at least 60 years old. $g \geq 60$
2. Conner's grade is above an 88%. $g > 88$
3. Nate was no more than 5 minutes late. $n \leq 5$
4. Kara's ring finger is smaller than a size 7. $r < 7$
5. Mandi needs to arrive between 5 and 6 o'clock. $5 < m < 6$

Try These

6. The temperature is less than 32° F.
7. The forensics team must have at least ten students to complete.
8. An elevator can carry no more than 15 people.
9. The trip to the Outer Banks takes between 6 and 8 hours.
10. To attend a Raven's game you will spend a minimum of $75.
11. The cost of dinner was more than $40.
12. Michael Jordan has a vertical leap of no less than 38 inches.
13. The number of kids that signed up to play tennis is at most 50.

Example

The Dallastown High School band wants to order polos. They will receive a discounted price of $24 per shirt plus a $25 delivery fee if they spend at least $250. What is the minimum number of polo shirts they must buy?

Let p = the number of polo shirts

Inequality: $24x + 25 \geq 250$

Solution: $x \geq 9.375$

Interpret your solution: The band must buy at least 10 shirts to receive the discount.

Try These

14. You want to rent a limousine for a trip to Baltimore's Inner Harbor. The limo costs $300 for the night and $0.15 per mile. You have a maximum of $350 to spend. What is the maximum number of miles you can travel?

    Let

    Inequality:

    Solution:

    Interpret your answer:

15. Taylor is having a birthday party. Chuck E. Cheese costs $13 per person while Suburban Bowlerama costs $10 per person plus a $24 fee for food. For how many people would Chuck E. Cheese be cheaper than going to Suburban Bowlerama?

    Let

    Inequality:

    Solution:

    Interpret your answer:

16. The senior class wants to make money for their senior party. They already have \$2500 raised and want to have more than \$4000. They decide to sell gift cards for Giant Foods. Giant will give them \$5 for every gift card they sell. What is the minimum number of gift cards the senior class must sell?
    Let
    Inequality:

    Solution:

    Interpret your answer:

17. George does not want to spend more than \$225 for Christmas presents. He has already spent \$176. He finds vintage t-shirts for \$12 that he would like to buy for his friends. What is the greatest number of t-shirts he can buy without exceeding his budget?
    Let
    Inequality:

    Solution:

    Interpret your answer:

18. Ellie needs to order books for her book club. Each book cost \$7.99. She can get free shipping if she spends more than \$100. What is the minimum number of books she needs to order to receive free shipping?
    Let
    Inequality:

    Solution:

    Interpret your answer:

Unit 6

Systems of Equations

Objectives:

1. Determine whether a point is a solution to a system of equations
2. Solve a system of equations by graphing (including infinite solutions, no solutions, and one solution)
3. Solve a system of equations using the substitution method
4. Solve a system of equations using the elimination method
5. Write, solve, and interpret systems of equations in the context of problem situations

Objectives I must do for Test B: (circle ones you must do)

1 2 3 4 5

Determining Whether a Point is a Solution to a System of Equations

How to test a point:

- To tell whether a point is a solution to a system, substitute the ordered pair into EACH equation for *x* and *y*.
- If the ordered pair satisfies BOTH equations, then it is a solution to the system.
- If the ordered pair does not work for either of the equations, it is NOT a solution.

Example

Is $(-1, 6)$ a solution to the system: $\begin{cases} x + 7 = y \\ -4x + 2 = y \end{cases}$

Substitute in values for both equations: $x = -1$ and $y = 6$.

Equation 1:	Equation 2:
$x + 7 = y$	$-4x + 2 = y$
$-1 + 7 \;\; ? \;\; 6$	$-4(-1) + 2 \;\; ? \;\; 6$
$6 = 6$	$4 + 2 \;\; ? \;\; 6$
Point works for Equation 1	$6 = 6$ → point works for Equation 2

Since the point makes BOTH equations true, the point $(-1, 6)$ IS a solution to the system.

Try These

Determine if the point is a solution to the system. You must show work to support your answer.

1.) Is $(-5, -3)$ a solution to the system:

$$\begin{cases} y = -2(x - 4) - 7 \\ y = (x - 4) - 4 \end{cases}$$

2.) Is $(-4, -2)$ a solution to the system:

$$\begin{cases} y = -3(x + 1) + 3 \\ 4x + 2y = 2 \end{cases}$$

3.) Is $(3, 1)$ a solution to the system:

$$\begin{cases} y = -2(x - 2) + 3 \\ y = -3x + 10 \end{cases}$$

4.) Is $(-3, -11)$ a solution to the system:

$$\begin{cases} -4x + y = 1 \\ y = x - 2 \end{cases}$$

5.) Is $(3, -10)$ a solution to the system:

$$\begin{cases} y = -4x - 2 \\ 3x + y = -1 \end{cases}$$

6.) Is $(1, -3)$ a solution to the system:

$$\begin{cases} y = 3(x+2) - 8 \\ y = 2(x+2) - 7 \end{cases}$$

7.) Is $(3, 3)$ a solution to the system:

$$\begin{cases} y = 4(x-3) + 3 \\ y = -3x + 12 \end{cases}$$

8.) Is $(4, 4)$ a solution to the system:

$$\begin{cases} y = 3(x-5) + 7 \\ -x + y = 0 \end{cases}$$

9.) Is $(4, -2)$ a solution to the system:

$$\begin{cases} y = -3(x-2) + 5 \\ y = 2(x-2) \end{cases}$$

10.) Is $(-6, -3)$ a solution to the system:

$$\begin{cases} 4x - 2y = -18 \\ y = 3x + 12 \end{cases}$$

Solving a System of Equations by Graphing

To solve a system of equations by graphing you need to ***graph each line separately***.
You will get one of the following answers:

- If the lines **intersect**, the solution is their **intersection point**
- If the lines are **parallel**, there is **no solution**
- If the lines are the **same line**, the answer is **all points on the line**

Example

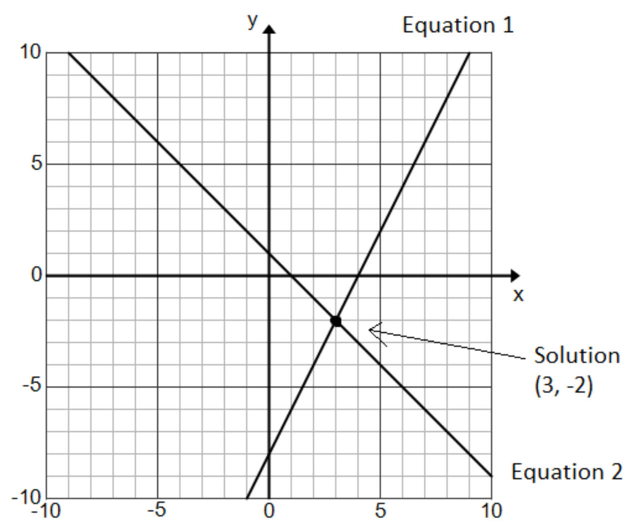
Solve the following system by graphing: $\begin{cases} 2x - y = 8 \\ x + y = 1 \end{cases}$

Step 1: Graph the first equation. You might need to convert to slope-intercept form (solve for y)

$$2x - y = 8$$
$$\underline{-2x \qquad\quad - 2x}$$
$$\underline{-y = -2x + 8}$$
$$-1 \quad\; -1 \quad -1$$

$\boldsymbol{y = 2x - 8}$ **→ Graph this equation**

(Slope = $\boldsymbol{2}$, $\boldsymbol{y}$-int = $\boldsymbol{-8}$)



Step 2: Graph the second equation. You might need to convert to slope-intercept form.

$$x + y = 1$$
$$\underline{-x \qquad\quad - x}$$

$\boldsymbol{y = -x + 1}$ **→ Graph this equation**

(Slope = $\boldsymbol{-1}$, $\boldsymbol{y}$-int = $\boldsymbol{1}$)

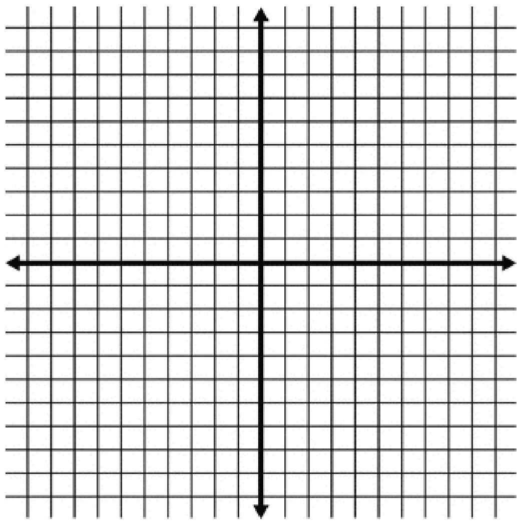
Step 3: Find the solution. Since these lines intersect, the solution is the point where they intersect.

Solution: $\boldsymbol{(3, -2)}$

Practice

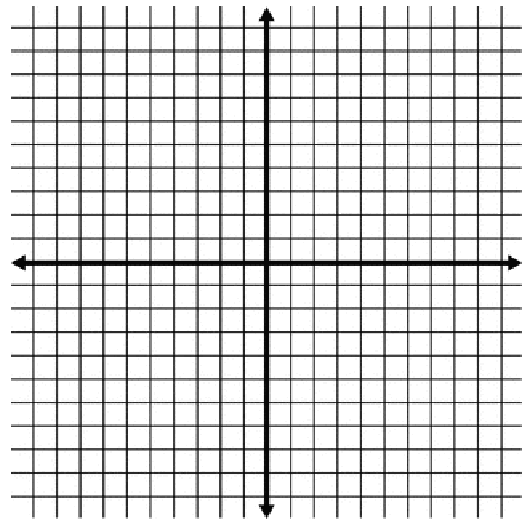
Solve each system by graphing.

1.) $\begin{cases} y = -x + 1 \\ y = x - 3 \end{cases}$



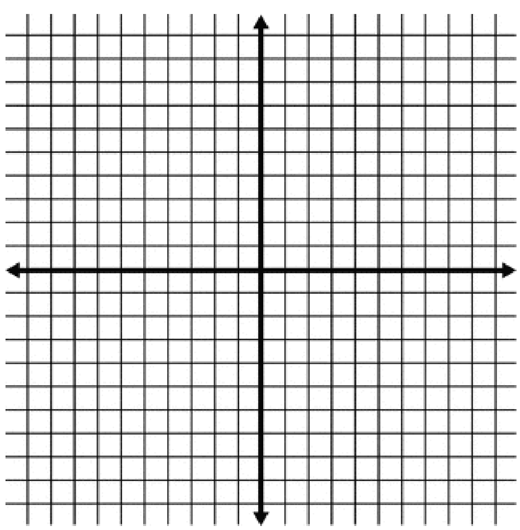
Solution: ______________________

2.) $\begin{cases} 3x + y = 4 \\ -3x + y = -2 \end{cases}$



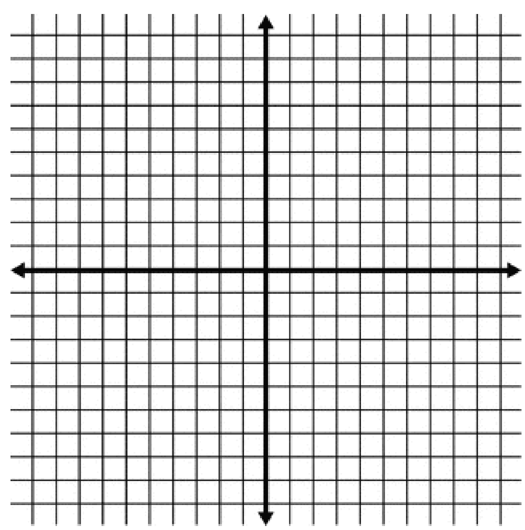
Solution: _________________________

3.) $\begin{cases} x + y = -2 \\ y = 2x + 7 \end{cases}$



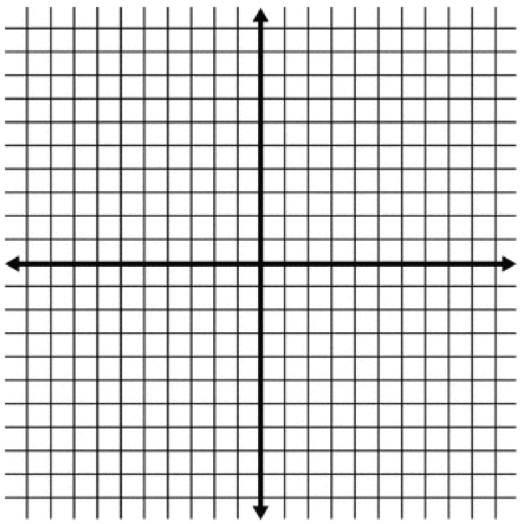
Solution: ______________________

4.) $\begin{cases} y = -3x + 1 \\ -4x + 2y = 2 \end{cases}$



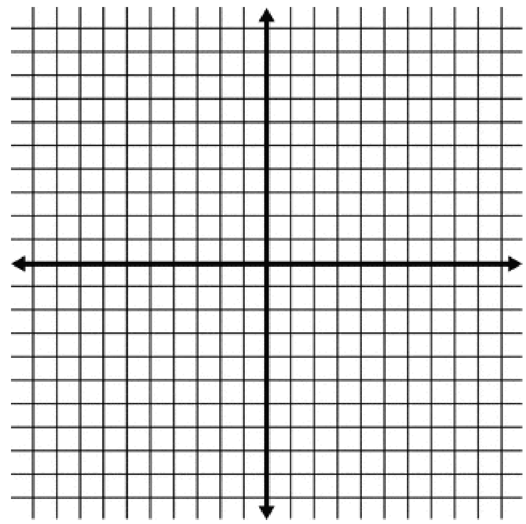
Solution: _________________________

5.) $\begin{cases} y = -x - 3 \\ y = x - 1 \end{cases}$



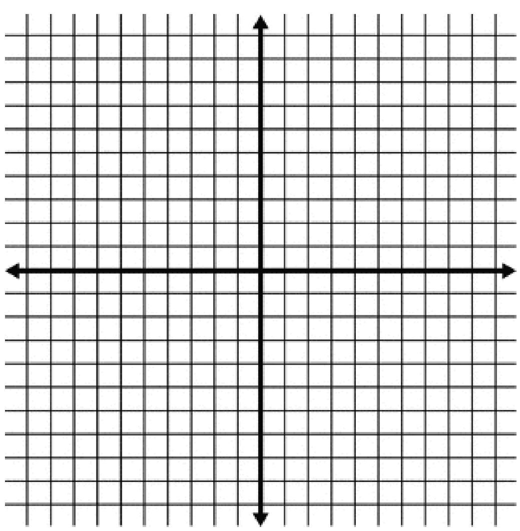
Solution: ______________________

6.) $\begin{cases} y = -2x + 3 \\ y = 3x + 3 \end{cases}$



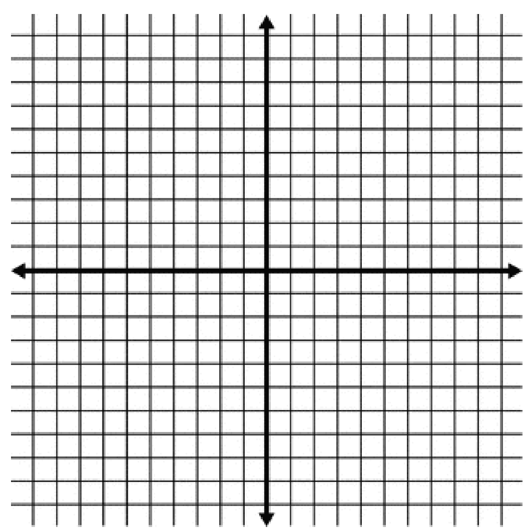
Solution: _________________________

7.) $\begin{cases} x = 2 \\ y = x + 1 \end{cases}$



Solution: ______________________

8.) $\begin{cases} y = -2 \\ 2x - 2y = -4 \end{cases}$



Solution: _________________________

Unit 6 Objective 3 Remediation

Solving a System of Equations using Substitution Method

Steps for Substitution

1. Solve one of the equations for one of its variables → need to have either $x =$ or $y =$
2. Substitute the expression from step 1 into the OTHER equation and solve for the other variable.
3. Substitute the value from step 2 into the one of the original equations and find the value of the second variable.

Example

Solve the following system using substitution: $\begin{cases} 2x - y = 6 \\ 2x + 2y = -9 \end{cases}$

Step 1: Solve one of the equations for one of its variable. Look for the variable that has a coefficient of 1 (no number written in front). We will be solving the first equation for y.

Equation 1: $2x - y = 6$ → solve for y

$$\underline{-2x \qquad -2x}$$

$$\frac{-y}{-1} = \frac{-2x}{-1} + \frac{6}{-1}$$ → Divide by -1 to get y all by itself

$y = 2x - 6$ → y is by itself so we are ready for Step 2

Step 2: Take the equation from Step 1 ($y = 2x - 6$) and substitute $2x - 6$ wherever you see y written in the OTHER equation

Equation 2: $2x + 2y = -9$ ← Write $2x - 6$ instead of y

$2x + 2(2x - 6) = -9$ ← Distribute

$2x + 4x - 12 = -9$ ← Solve for x

$6x - 12 = -9$

$$\underline{\quad +12 \quad +12}$$

$$\frac{6x}{6} = \frac{3}{6}$$

$$x = \frac{1}{2}$$

Step 3: Substitute $\frac{1}{2}$ in for x and solve for y. It does not matter which equation you use, but it will be easier to use the equation from step 1 since we already have $y =$

$y = 2x - 6$ → $y = 2\left(\frac{1}{2}\right) - 6$ → $y = 1 - 6$ → $y = -5$

Solution: $\left(\frac{1}{2}, -5\right)$

Practice

Solve each system using the substitution method.

1.) $\begin{cases} x - 3y = -1 \\ y = -4x + 22 \end{cases}$

2.) $\begin{cases} -3x - 4y = 12 \\ y = -2x - 3 \end{cases}$

3.) $\begin{cases} -3x - 4y = 0 \\ x = -2y + 2 \end{cases}$

4.) $\begin{cases} 2x + y = -2 \\ 5x - 2y = 4 \end{cases}$

5.) $\begin{cases} y = -4x - 1 \\ y = 8x + 11 \end{cases}$

6.) $\begin{cases} 3x + 2y = 10 \\ x = y \end{cases}$

7.) $\begin{cases} x = -4y + 4 \\ x = 2y + 10 \end{cases}$

8.) $\begin{cases} 12x + 3y = 6 \\ y = -4x + 19 \end{cases}$

Unit 6 Objective 4 Remediation

Solving a System of Equations using Elimination Method

Steps for Elimination

1. Multiply one or both of the equations by a number so that the coefficients of one of the variables are opposites
2. Add the equations from step 1 together. Combining the equations will eliminate one of the variables.
3. Solve for the remaining variable.
4. Substitute the value from step 3 into the one of the original equations and find the value of the second variable.

Example

Solve the following system using Elimination: $\begin{cases} 9x + 2y = 0 \\ 3x - 5y = 17 \end{cases}$

Step 1: We need to use multiplication to get the same number, different signs in front of one of the variable. In this example, we will multiply the bottom equation by -3 so the x will then have -9 in front of it and will eliminate with the $9x$ in the first equation.

$\begin{cases} 9x + 2y = 0 \\ 3x - 5y = 17 \quad (-3) \end{cases}$ ← Multiply ALL terms in the bottom equation by -3

New Equation: $-9x + 15y = -51$
1st Equation (did not change): $9x + 2y = 0$

Step 2 and 3: ADD the equations together. The x variables will get eliminated. Then solve for y.

$$\begin{array}{r} -9x + 15y = -51 \\ \underline{+ \quad 9x + 2y \; = \quad 0} \\ 17y = -51 \end{array}$$ ← Solve for y

$$y = -3$$

Step 4: Substitute -3 in for y and solve for x. It does not matter which equation you use.

Equation 1: $9x + 2y = 0$ ← substitute -3 in for y and solve for x

$$9x + 2(-3) = 0 \rightarrow \underline{9x - 6 = 0}$$
$$+6 \quad +6$$
$$\underline{9x = 6}$$
$$9 \quad 9$$
$$x = \frac{2}{3}$$

Solution: $\left(\frac{2}{3}, -3\right)$

Special Cases

When you add the two equations together and BOTH variables get eliminated, the problem is a special case and the answer is either No Solution or All Points on the Line (Infinite).

No Solution	**All Points on the Line (Infinite Solutions)**
$x - y = -2$ $\underline{-x + y = 5}$ $\mathbf{0 = 3}$ ← **This is a FALSE statement** **The answer is NO SOLUTION**	$2x + y = -5$ $\underline{-2x - y = 5}$ $\mathbf{0 = 0}$ ← **This is a TRUE statement** **The answer is ALL POINTS ON THE LINE**

Practice

Solve each system using the Elimination Method.

1.) $\begin{cases} x + y = -3 \\ x - 4y = -8 \end{cases}$

2.) $\begin{cases} -2x - y = 11 \\ -2x - 3y = 21 \end{cases}$

3.) $\begin{cases} 3x + 4y = 10 \\ 6x + 8y = -20 \end{cases}$

4.) $\begin{cases} 4x + 2y = -20 \\ 4x - 3y = -10 \end{cases}$

5.) $\begin{cases} 2x - 4y = 6 \\ 4x - y = 5 \end{cases}$

6.) $\begin{cases} 3x + 4y = -4 \\ 2x - y = 1 \end{cases}$

7.) $\begin{cases} x - 4y = -20 \\ 4x - 3y = -28 \end{cases}$

8.) $\begin{cases} x - 3y = 8 \\ -2x + 6y = 10 \end{cases}$

9.) $\begin{cases} 3x - 5y = 13 \\ -4x - 2y = 26 \end{cases}$

10.) $\begin{cases} 3x - 4y = 16 \\ -2x - 3y = 12 \end{cases}$

Unit 6 Objective 5 Remediation

Applications of Systems of Equations

Steps in Writing and Solving a System of Equations:

- Read and underline key words
- Identify the variables (Let $x =$; $y =$)
- Write a system of equations that models the problem, using keywords from the problem
  Remember: 2 Unknowns means 2 Equations
- Solve the system – use either Elimination or Substitution
- Write a sentence interpreting your answer

Practice

1.) Ian bought flowers and candy boxes. Flowers costs $9 each while candy boxes are only $8 each. If Ian spent $168 and only ended up with 19 items, how many flowers and candy boxes did Ian buy?

Let $x =$
$y =$

Equations (you need 2 equations since you have 2 unknowns)

Solution: ________________ Interpret Answer:

2.) Kate asked a bank teller to cash a $180 check using only $20 bills and $10 bills. The teller gave her a total of 11 bills. How many of each bill was Kate given?

Let $x =$
$y =$

Equations:

Solution: ________________ Interpret Answer:

3.) Sam bought Tshirts and socks. Tshirts are on sale at $10 each while socks are currently sold at $8 each. If Sam spent $172 and only got 20 items, how many Tshirts and socks did Sam buy?

Let $x =$

$y =$

Equations:

Solution: ________________ Interpret Answer:

4.) A caterer's total cost for catering a party includes a fixed cost, which is the same for every party. In addition the caterer charges a certain amount for each guest. If it costs $300 to serve 25 guests and $420 to serve 40 guests, find the fixed cost and the cost per guest.

Let $x =$

$y =$

Equations:

Solution: ________________ Interpret Answer:

5.) If you buy six pens and one mechanical pencil, you'll get only $1 change from your $10 bill. But if you buy four pens and two mechanical pencils, you'll get $2 change. How much does each pen and each pencil cost?

Let $x =$
$y =$

Equations:

Solution: ________________ Interpret Answer:

6.) A car rental agency charges a daily fee plus a cost per mile. If a car driven 40 miles in one day cost $28, and the same car driven 100 miles in one day cost $37, what are the daily fee and the cost per mile?

Let $x =$
$y =$

Equations:

Solution: ________________ Interpret Answer:

Unit 7

Systems of Inequalities

Objectives:

1. Graph the solution to a linear inequality
2. Solve a system of inequalities by graphing
3. Determine whether a point is a solution to a system of inequalities
4. Write a linear inequality or system of linear inequalities given a graph
5. Write, solve, and interpret systems of inequalities in the context of problem situations

Objectives I must do for Test B: (circle ones you must do)

1 2 3 4 5

Unit 7 Objective 1 Remediation

Graphing a Linear Inequality in Two Variables

Steps to Graph an Inequality

- Write the inequality in slope-intercept form (get y by itself)
- Graph the line $y = mx + b$ as a solid or dashed line
  - **Solid** if the inequality symbol is $\leq$ or $\geq$
  - Dashed if the inequality symbol is $<$ or $>$
- Pick a point to test that is **NOT** on the line
  - If the point creates a **true** statement, shade the side of the line where the point is located
  - If the point creates a **false** statement, shade the OTHER side of the line where the point is not

Example

Graph the solution of the inequality $5x + 2y < 6$.

<table>
<tr>
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- Change the inequality to slope-intercept form by solving the inequality for y
- Remember to switch the inequality sign if multiplying or dividing by a negative number

</td>
<td>

$$5x + 2y < 6$$
$$2y < -5x + 6$$
$$\frac{2y}{2} < \frac{-5x + 6}{2}$$
$$\boldsymbol{y < -\frac{5}{2}x + 3}$$

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- Graph the line $y = -\frac{5}{2}x + 3$
- Draw a dashed line because the inequality sign is $<$

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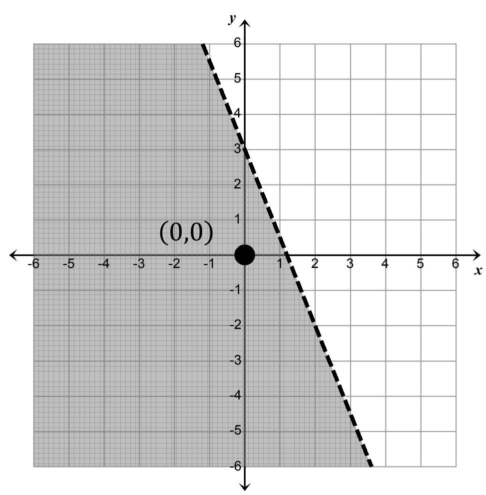

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- Pick a point to test that is not on the line
- Pick $(0, 0)$ and substitute it into the original inequality

$$5(0) + 2(0) < 6$$
$$0 < 6$$

- This is true so shade the side of the line of where $(0, 0)$ is located

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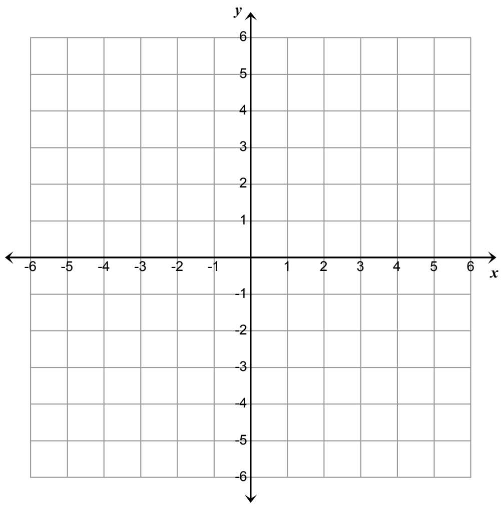
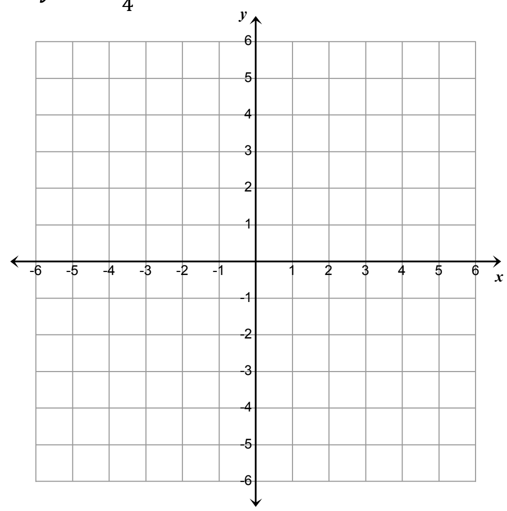
Graph the solution of the following inequalities.

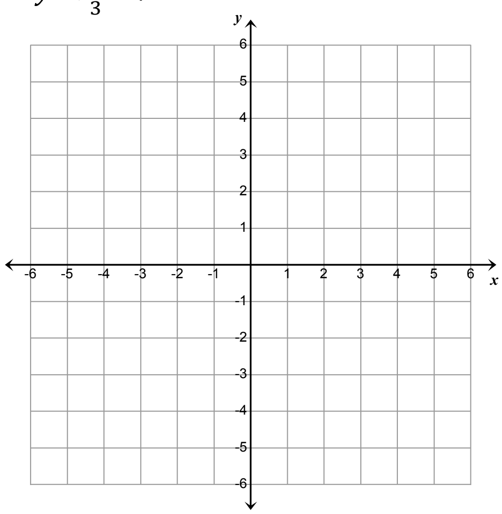
1. $y \leq x + 3$



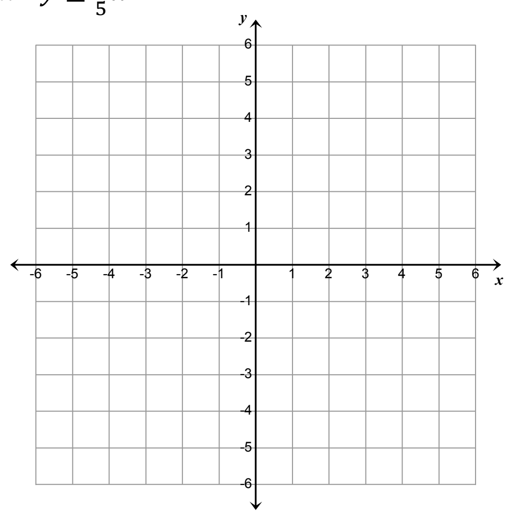
2. $y \geq -\frac{3}{4}x - 1$



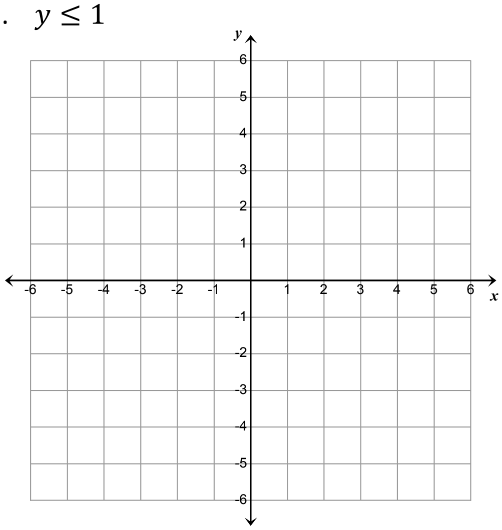
3. $y < \frac{2}{3}x + 2$



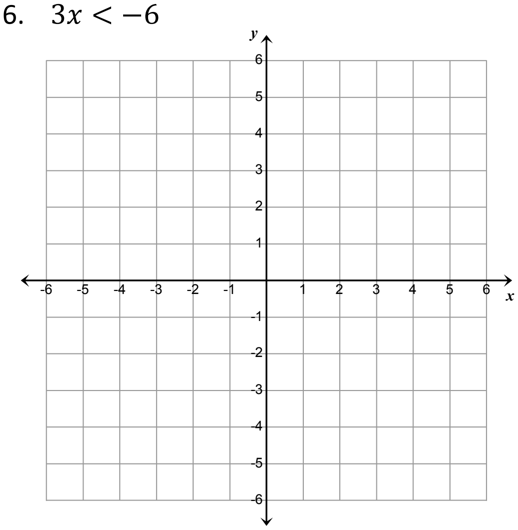
4. $y \leq \frac{2}{5}x$



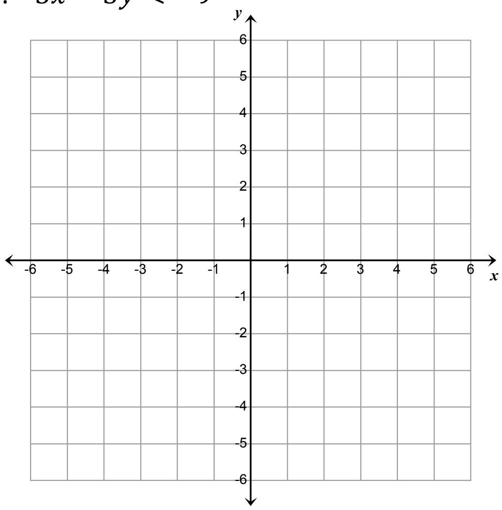
5. $y \leq 1$



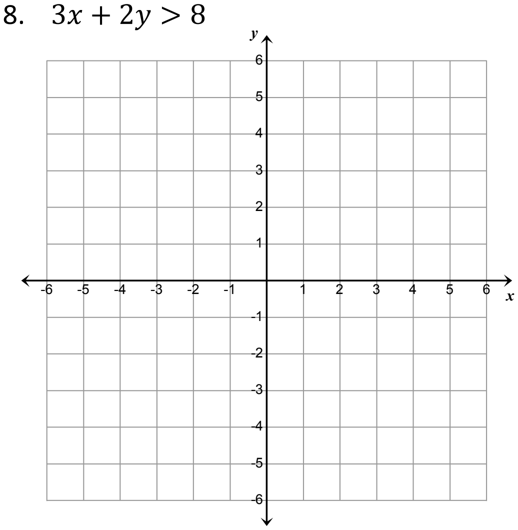
6. $3x < -6$



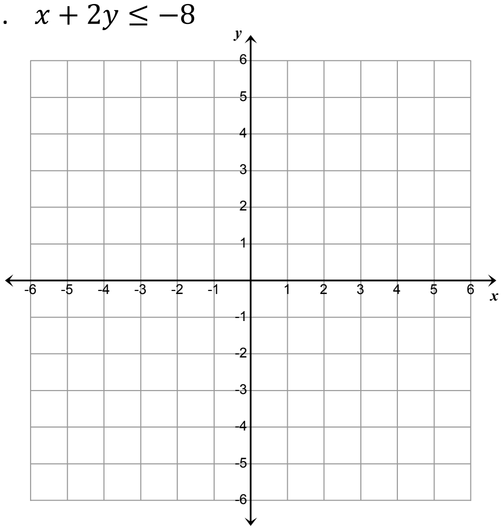
7. $5x - 3y < -9$



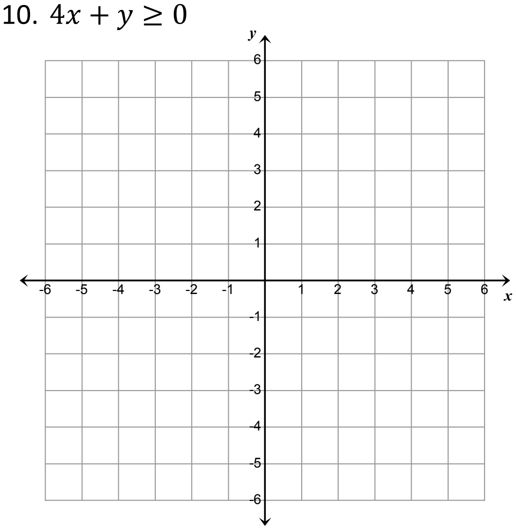
8. $3x + 2y > 8$



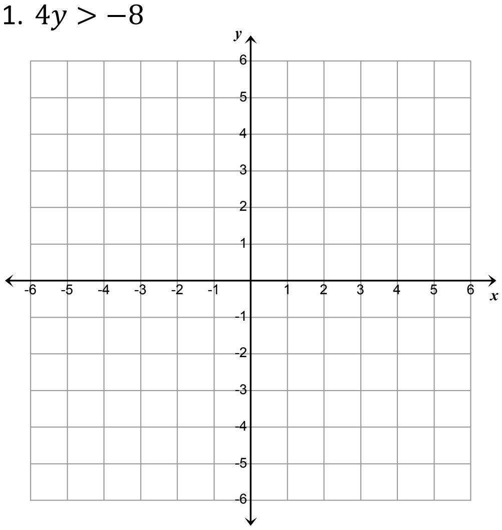
9. $x + 2y \leq -8$



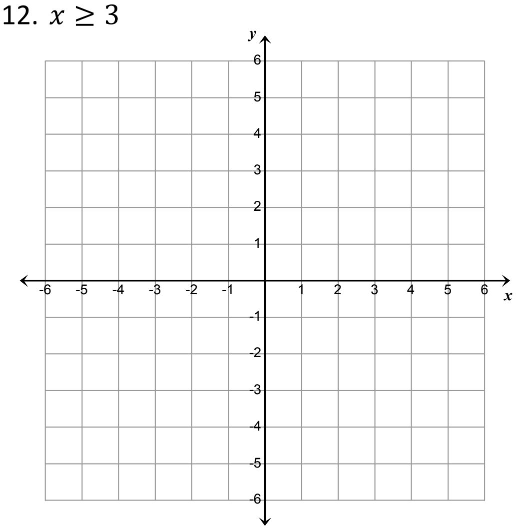
10. $4x + y \geq 0$



11. $4y > -8$



12. $x \geq 3$



Unit 7 Objective 2 Remediation

Graphing Systems of Linear Inequalities

Steps in Graphing Linear Inequalities

Before you begin, make sure both inequalities are in slope-intercept form. If they are not, change them to this form by solving for y.

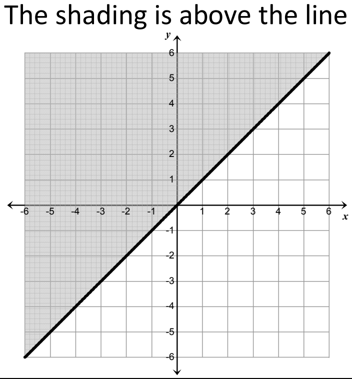
Step #1: Graph the first inequality; select a point to help determine which side to shade.
Step #2: Graph the second inequality; select a point to help determine which side to shade.
*Remember to shade in a different color or in a different direction
Step #3: Determine the solution. The solution is the **intersection of the shaded area.**
Step #4: Check a point – select a point that is in the solution and make sure it is a solution to all of the inequalities in the system.

Example

Graph the system of inequalities: $\begin{cases} y \geq x \\ y \geq -x+1 \end{cases}$

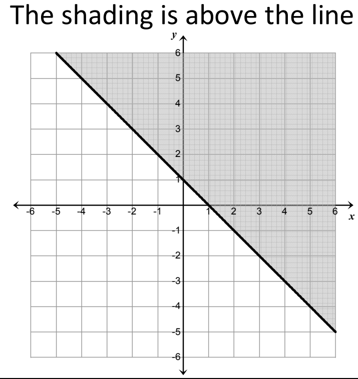
Graph the first inequality: $y \geq x$

- The line is solid because the inequality sign is $\geq$
- The shading is above the line

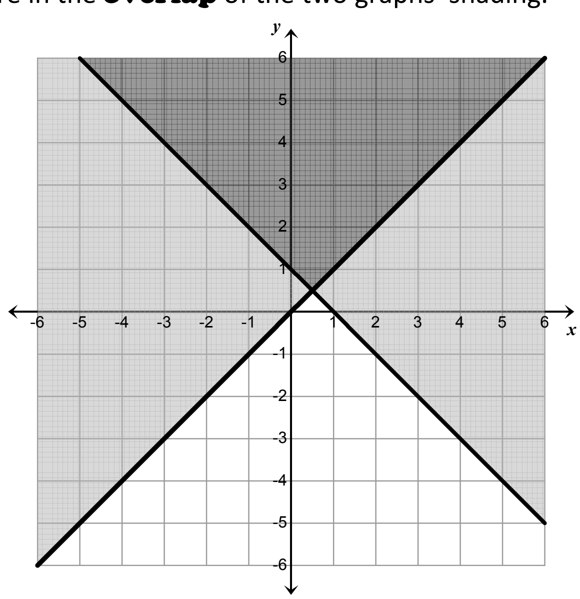


Graph the second inequality: $y \geq -x+1$

- The line is solid because the inequality sign is $\geq$
- The shading is above the line

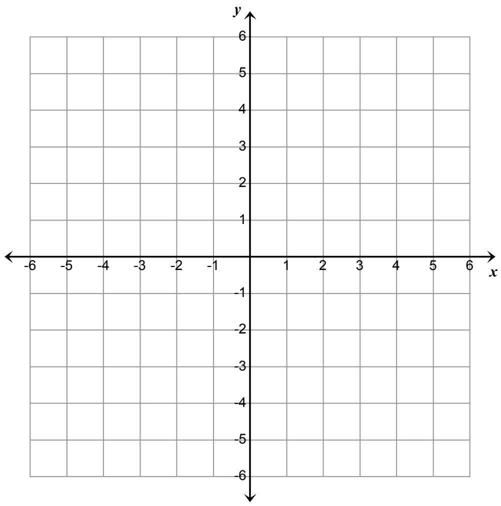


The solutions to the system are in the **overlap** of the two graphs' shading.

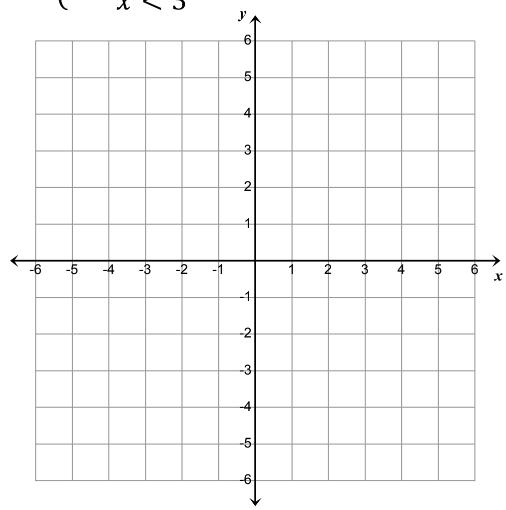


Graph the following systems of inequalities.

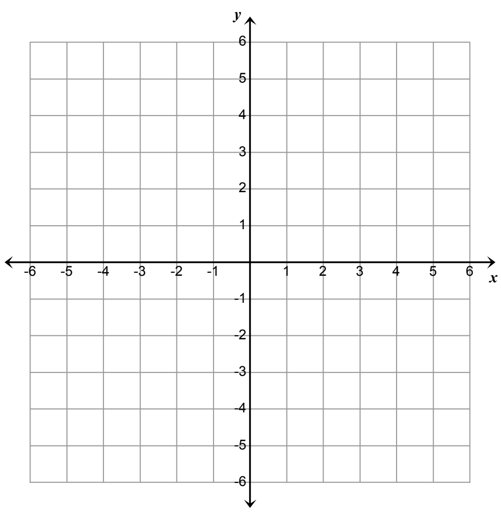
1. $\begin{cases} y \le x + 2 \\ y > 2x \end{cases}$



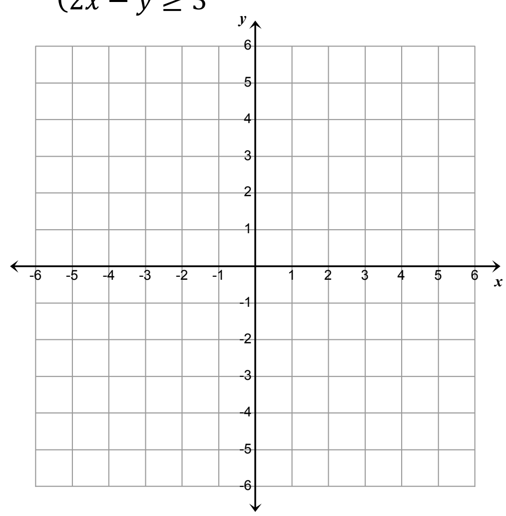
2. $\begin{cases} y \ge -\frac{1}{4}x - 1 \\ x < 3 \end{cases}$



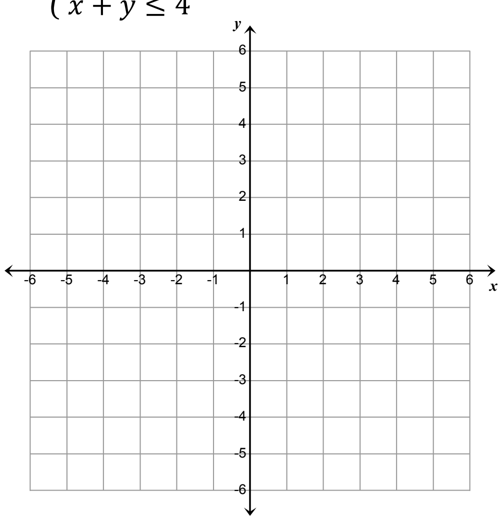
3. $\begin{cases} y \le 2x + 1 \\ y < x - 1 \end{cases}$



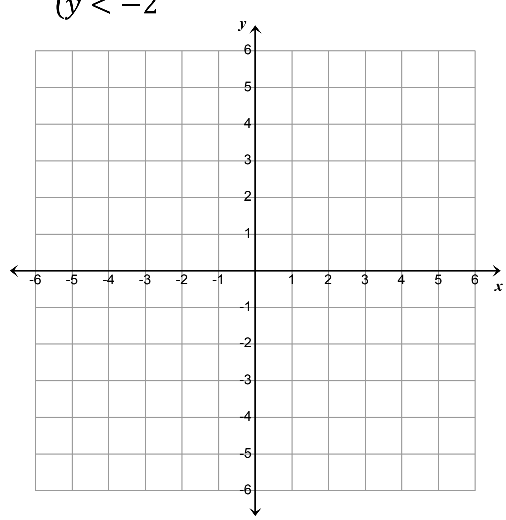
4. $\begin{cases} y \le \frac{2}{3}x \\ 2x - y \ge 3 \end{cases}$



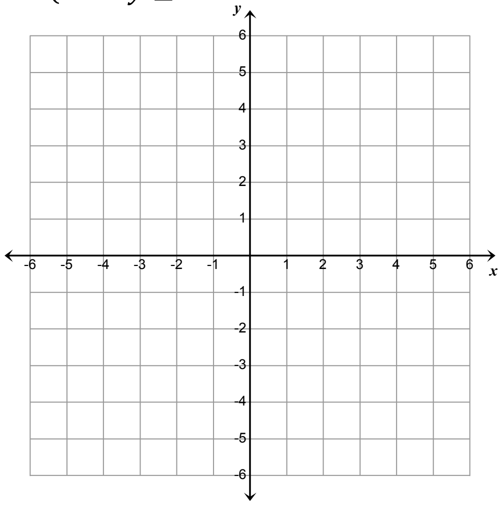
5. $\begin{cases} 3x + y > 6 \\ x + y \le 4 \end{cases}$



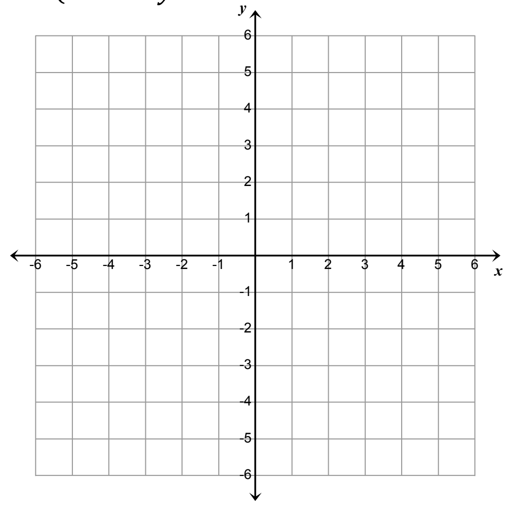
6. $\begin{cases} x < -1 \\ y < -2 \end{cases}$



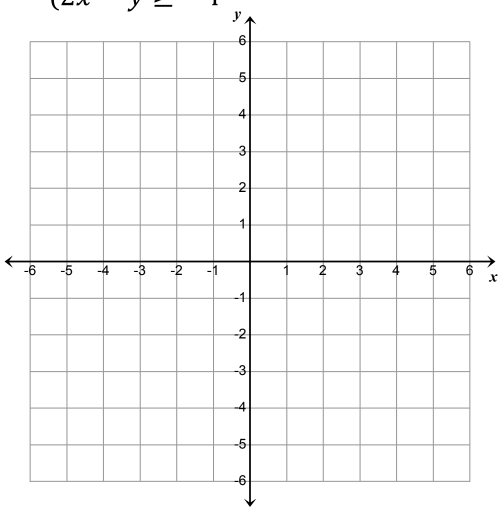
7. $\begin{cases} 2x - 3y \leq 3 \\ x + y \leq 5 \end{cases}$



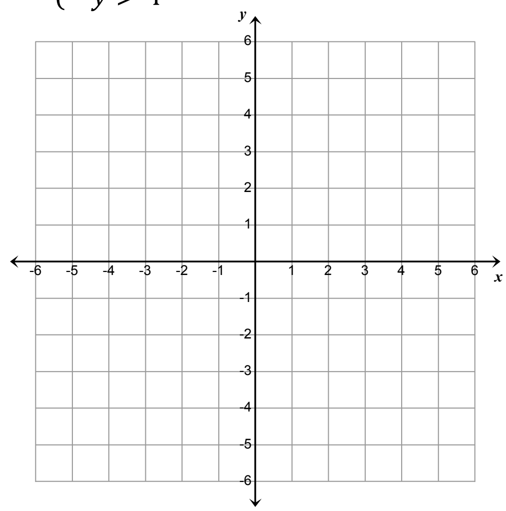
8. $\begin{cases} 2x + 3y < 15 \\ 3x - 2y > 0 \end{cases}$



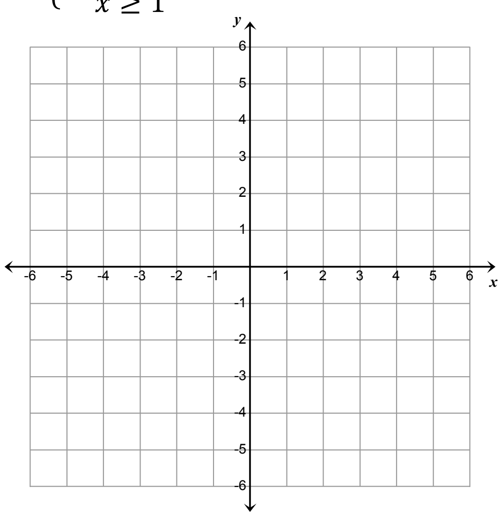
9. $\begin{cases} 2x - y \leq 3 \\ 2x - y \geq -4 \end{cases}$



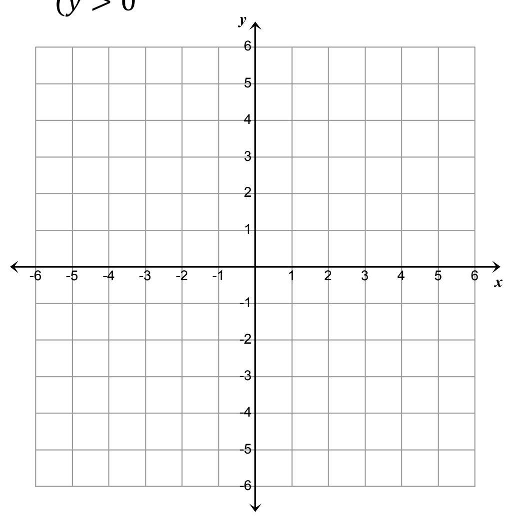
10. $\begin{cases} x + y > 0 \\ y > 4 \end{cases}$



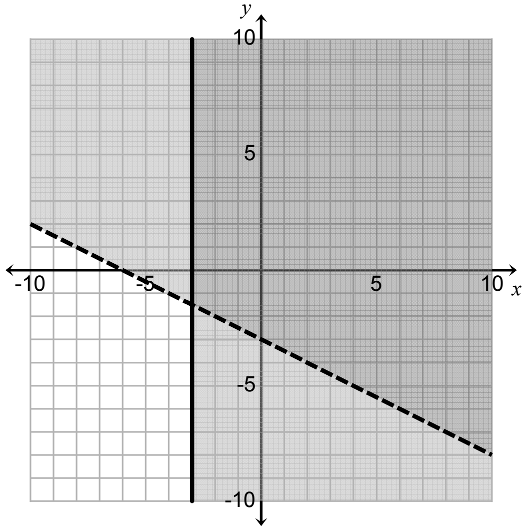
11. $\begin{cases} 3x + y < 1 \\ x \geq 1 \end{cases}$



12. $\begin{cases} y \geq x \\ y > 0 \end{cases}$



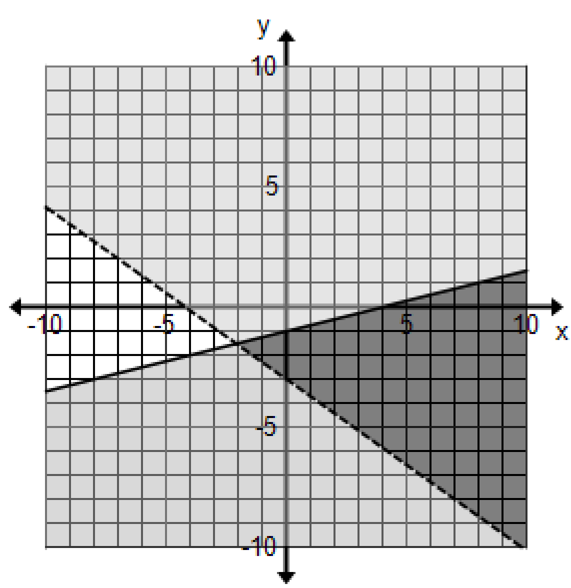
Determining if a Point is a Solution to a System of Linear Inequalities

Example 1	Example 2
Is the point **located in the overlap** of the shading? ➲ If yes, then the point is a solution. ➲ If no, then the point is not a solution.  Is the point $(1, -1)$ a solution? YES, it is located in the overlap. Is the point $(-3, -5)$ a solution? NO, it is not located in the overlap.	Does the point **make both inequalities true** when we plug in the x and y values? ➲ If yes, then the point is a solution. ➲ If no, then the point is not a solution. Is the point $(3, -1)$ a solution of the system $\begin{cases} y > -2x - 1 \\ x - 3y \leq 4 \end{cases}$? Plug in the values for x and y and see if you get a true statement. Do this for **both** inequalities. $-1 > -2(3) - 1$ $-1 > -7$ True $3 - 3(-1) \leq 4$ $6 \leq 4$ False The point $(3, -1)$ is NOT a solution because it does not make both inequalities true.

Try These

State whether or not the following points are solutions of the system of inequalities shown.

1. $(0, 0)$ ____________
2. $(0, -2)$ ____________
3. $(3, -3)$ ____________
4. $(-2, 2)$ ____________
5. $(4, 0)$ ____________
6. $(-8, -3)$ ____________
7. $(-3, 2)$ ____________
8. $(5, -6)$ ____________



Try These

Determine whether the point is a solution of the given system of inequalities.

9. Is the point $(3, 15)$ a solution to the system $\begin{cases} y > 3x + 2 \\ y < 4x - 3 \end{cases}$ 9. ____________

10. Is the point $(1, -2)$ a solution to the system $\begin{cases} x > 2y - 3 \\ y \leq -x + 6 \end{cases}$ 10. ____________

11. Is the point $(-3, -5)$ a solution to the system $\begin{cases} 3y - 2x > 5 \\ 4x + y \geq 9 \end{cases}$ 11. ____________

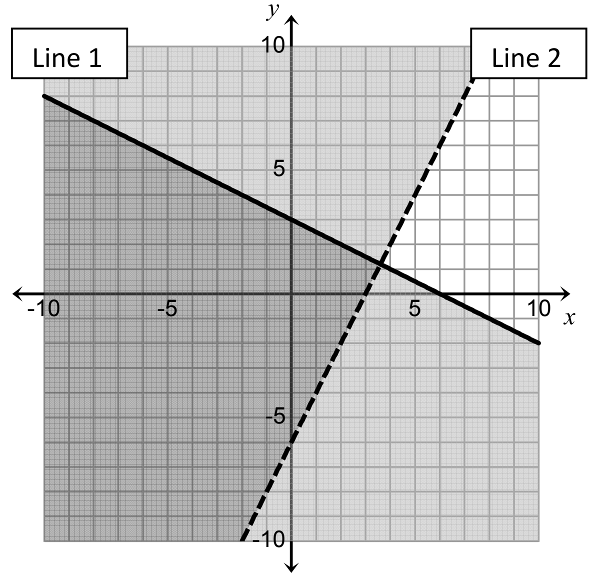
12. Is the point $(1, 3)$ a solution to the system $\begin{cases} 2x - 7y < 5 \\ 11 > 3y + x \end{cases}$ 12. ____________

13. Is the point $(-4, -3)$ a solution to the system $\begin{cases} y \leq \frac{3}{2}x + 3 \\ x + y < 1 \end{cases}$ 13. ____________

14. Is the point $(-6, 8)$ a solution to the system $\begin{cases} y \geq \frac{3}{2}x - 2 \\ 5x + 3y > -6 \end{cases}$ 14. ____________

Writing a System of Inequalities from a Graph

Example: Write the system of inequalities given the graph.



LINE 1: Find the slope and y-intercept of line 1.

$m = -\frac{1}{2}$ and $b = 3$

Write the inequality y ? $-\frac{1}{2}x + 3$

Pick a point in the overlap to test. I'll pick $(0, 0)$.

0 ? $-\frac{1}{2}(0) + 3$

Put in the symbol that will make the statement true.

So, inequality #1 is $y \leq -\frac{1}{2}x + 3$

LINE 2: Find the slope and y-intercept of line 2.

$m = 2$ and $b = -6$

Write the inequality y ? $2x - 6$

Pick a point in the overlap to test. I'll pick $(0, 0)$.

0 ? $2(0) - 6$

Put in the symbol that will make the statement true.

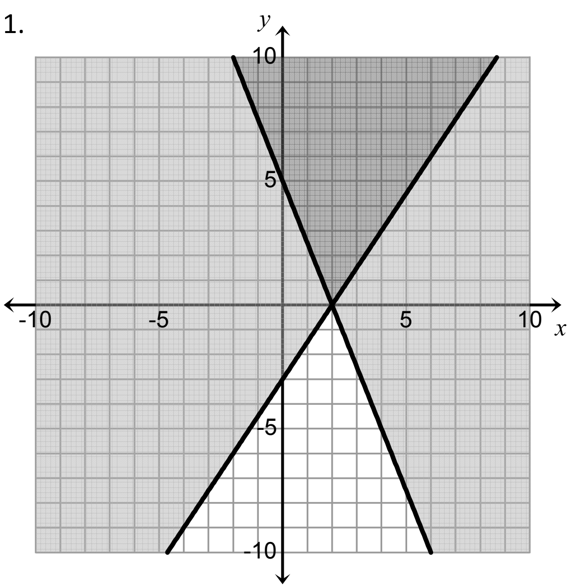
So, inequality #2 is $y > 2x - 6$

Answer: $\begin{cases} \boldsymbol{y \leq -\frac{1}{2}x + 3} \\ \boldsymbol{y > 2x - 6} \end{cases}$

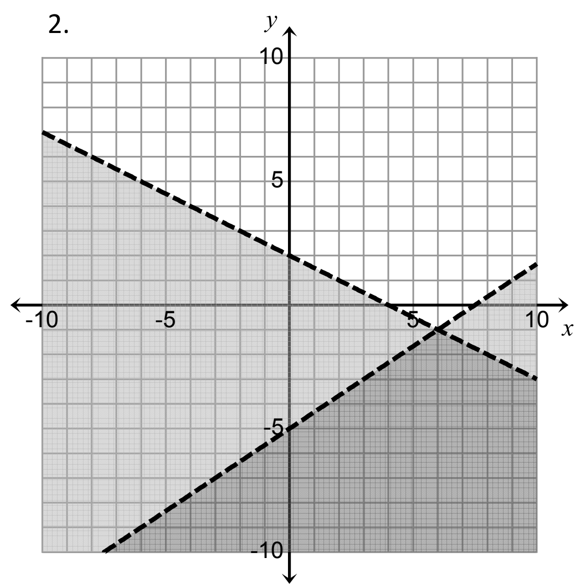
Try These

Write the system of inequalities given the graph.

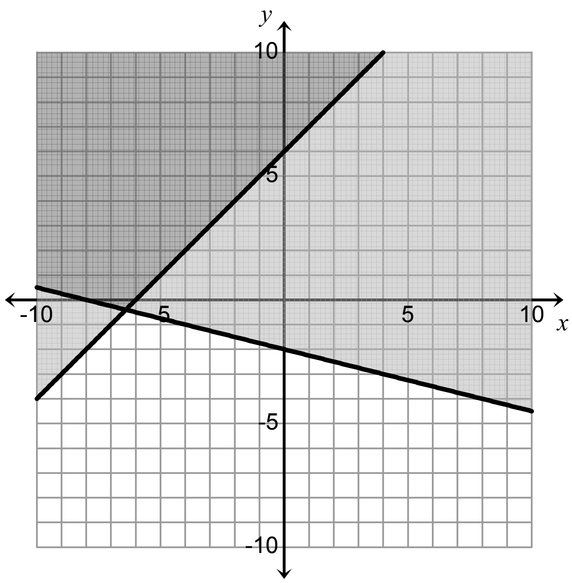
1.



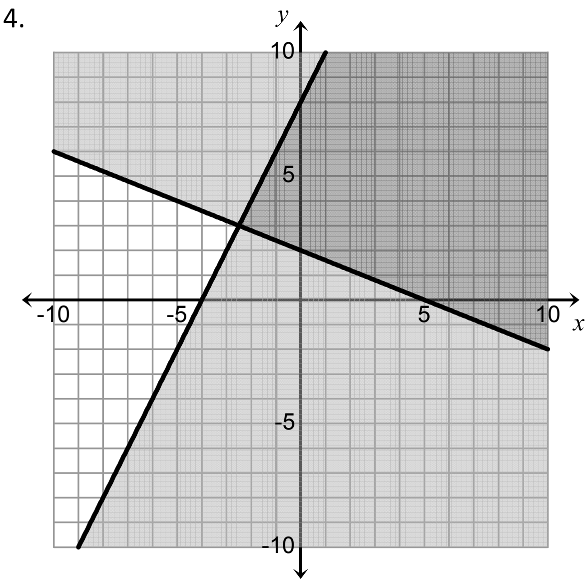
2.



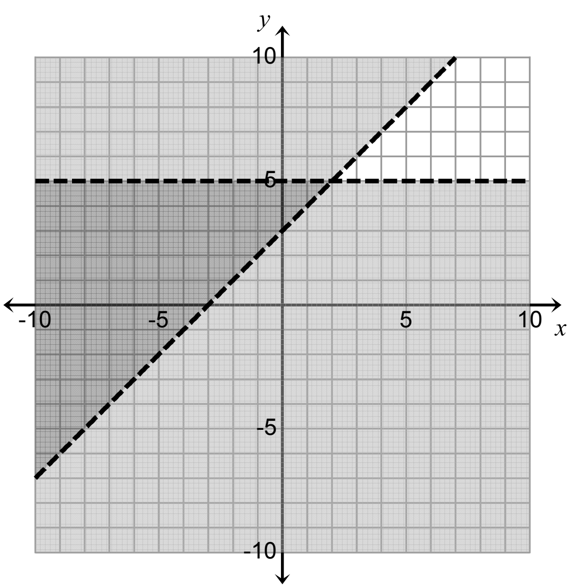
3.


y
10
5
-10
5
5
10
x
-5
-10

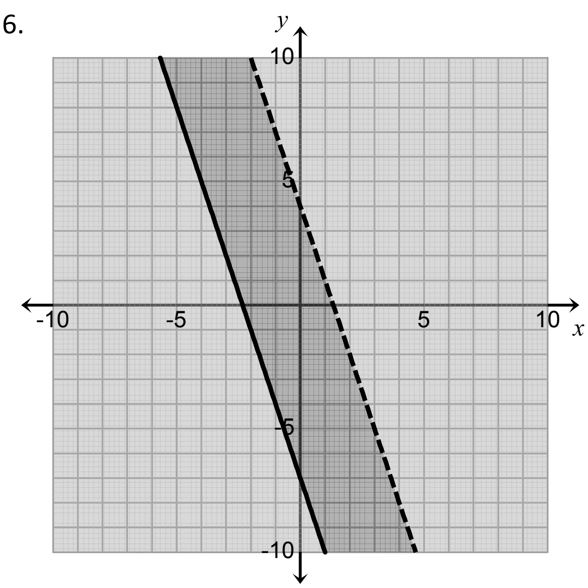
4.


y
10
5
-10
-5
5
10
x
-5
-10

5.


y
10
5
-10
-5
5
10
x
-5
-10

6.


y
10
5
-10
-5
5
10
x
-5
-10

Unit 7 Objective 5 Remediation

Write, Solve and Interpret a System of Linear Inequalities

Steps to Writing and Solving a System of Linear Inequalities

1. Read the problem and underline important information.
2. Identify the variables and define the unknowns they will represent.
3. Write two inequalities. The inequalities $x \geq 0$ and $y \geq 0$ should also be written.
4. Graph the two inequalities and shade the intersection.
5. Find a solution (point) that lies in the intersection and write your answer in a complete sentence.

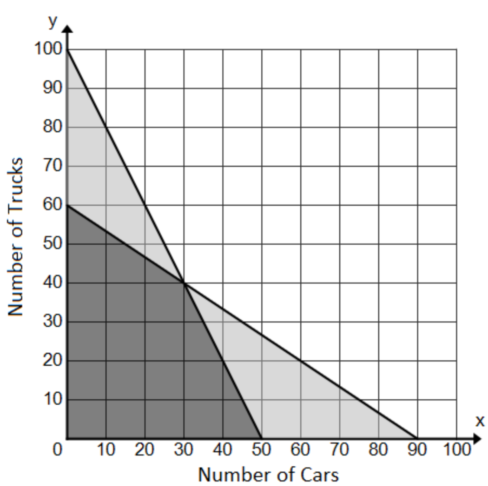
Example

Wildcat Toyz is a small toy company that specializes in toy cars and toy trucks. The people at Wildcat Toyz are confident they can sell all the toy cars and trucks they made. But there are two constraints that limit their production today:

Wheels: Each car needs 4 wheels. Each truck needs 6 wheels.
Wildcat Toys has no more than 360 wheels in stock

Seats: Each car needs 2 seats. Each truck needs 1 seat.
Wildcat Toys has no more than 100 seats in stock.

Write a system of inequalities to determine the possible production of cars and trucks at Wildcat Toyz.

<table>
<tr>
<td>Let $x =$ number of cars
Let $y =$ number of trucks</td>
<td>Inequality #1: $4x + 6y \leq 360$
Inequality #2: $2x + y \leq 100$
Inequality #3: $x \geq 0$
Inequality #4: $y \geq 0$</td>
</tr>
<tr>
<td>Graph the inequalities. The solution is the overlap of the shading.

</td>
<td>Give one possible combination of cars and trucks that Wildcat Toyz can produce today.
Pick any point in the overlap of the shading.
Possible answers are…
• $(20, 20)$ → 20 Cars and 20 Trucks
• $(30, 10)$ → 30 Cars and 10 Trucks
• $(10, 50)$ → 10 Cars and 50 Trucks
• $(0, 60)$ → 0 Cars and 60 Trucks</td>
</tr>
</table>

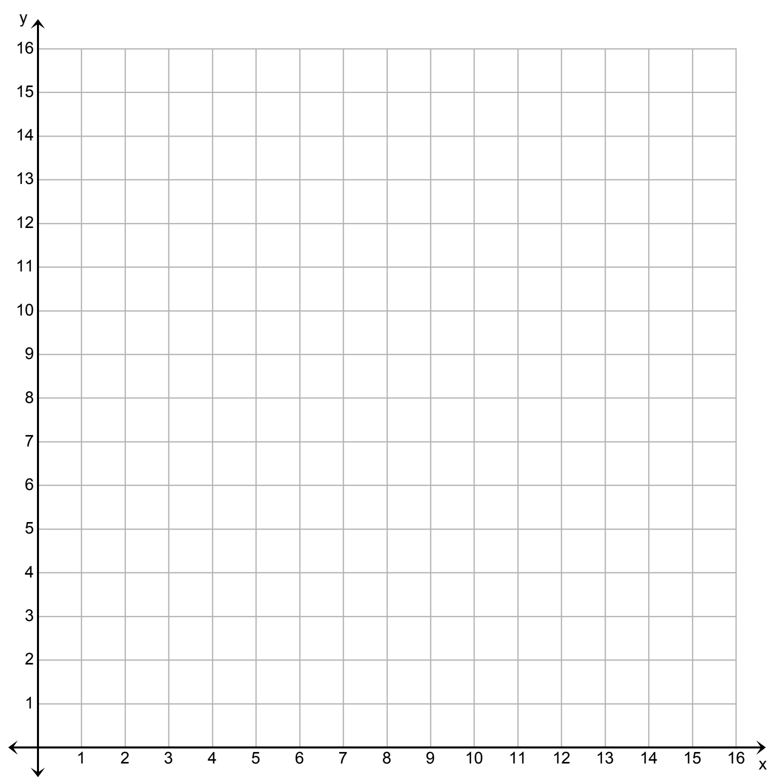
Write the inequalities that would model each situation and solve.

1. You are fishing in a lake for salmon and trout. You can sell the salmon for $3 each and trout for $5 each. Regulations say that you can't catch more than 15 fish a day, and you can't catch more than $55 of fish a day. Write and solve a system of inequalities representing the amount of fish you could catch a day.

   Let x =

   Let y =

   Inequalities:



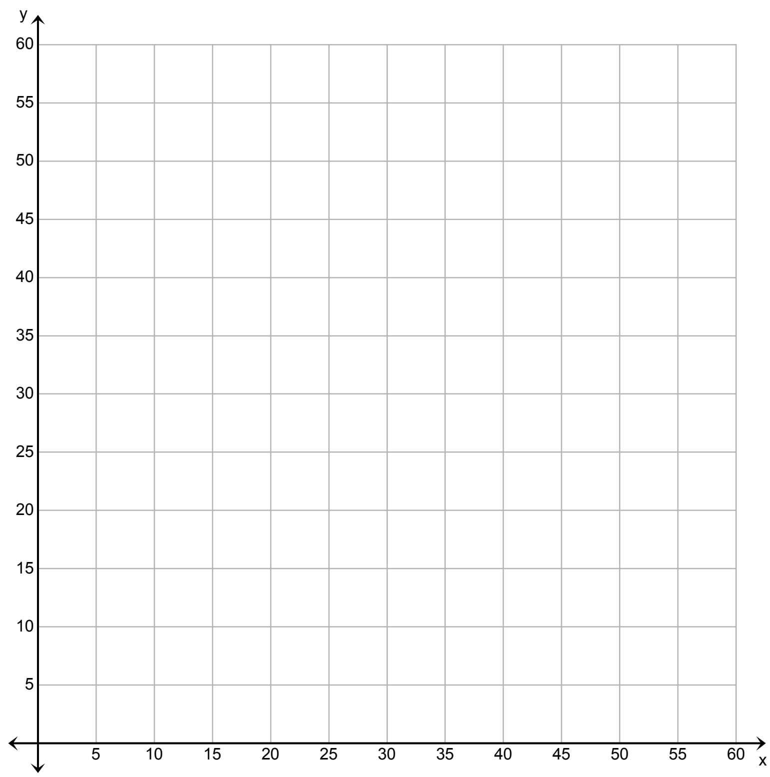
   Give one solution that would satisfy this situation:

2. You are shopping for baseballs and tennis balls at a sports store. Each baseball costs \$3 and each tennis ball costs \$2. You want to buy not fewer than 45 baseballs and tennis balls altogether, and you have a \$100 budget. Write and solve a system of inequalities representing the number of balls you could buy.

   Let x =

   Let y =

   Inequalities:



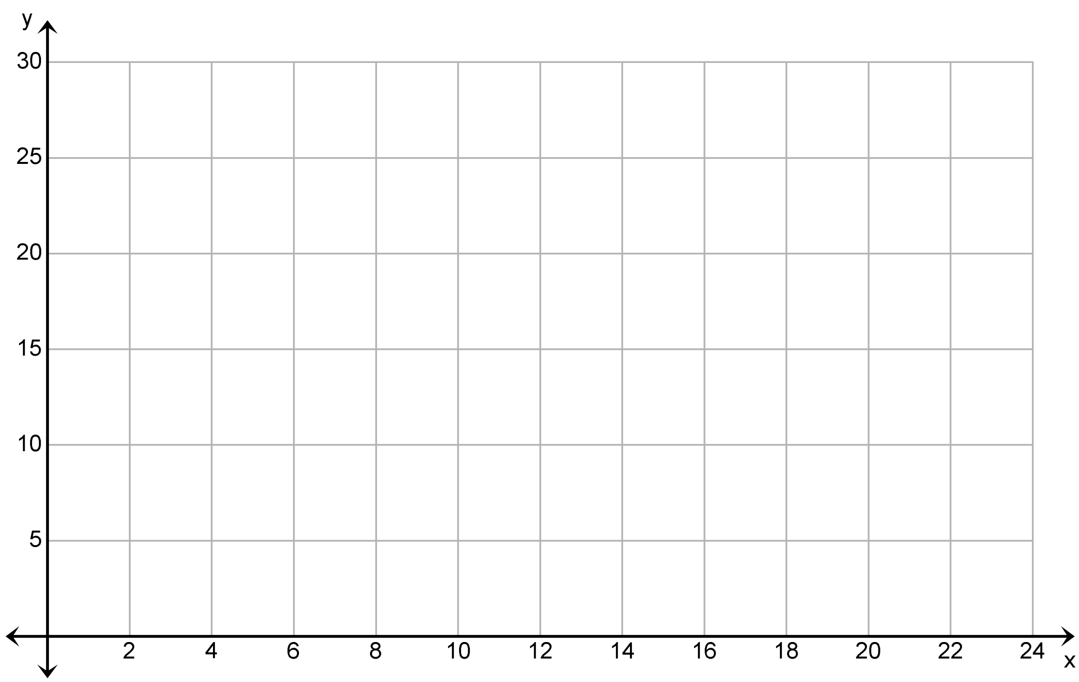
   Give one solution that would satisfy this situation:

3. You are shopping for notebooks and pens at an office supply store. Each notebook costs \$4 and each pack of pens costs \$2. You don't want to buy more than 20 notebooks and pens together, and you have a \$50 budget. Write and solve a system of inequalities representing the number of office supplies you could buy.

   Let x =

   Let y =

   Inequalities:



   Give one solution that would satisfy this situation:

Unit 8

Exponents

Objectives:

1. Simplify exponents using the Product-of-Powers Property
2. Simplify exponents using the Power-of-a-Power Property
3. Simplify exponents using the Power-of-a-Product Property
4. Simplify exponents using the Quotient-of-Powers Property
5. Simplify exponents using the Power-of-a-Quotient Property
6. Simplify exponents using the Zero Exponent Property
7. Simplify exponents using the Negative Exponent Property
8. Simplify exponents using multiple properties

Objectives I must do for Test B: (circle ones you must do)

1 2 3 4 5 6 7 8

Product of Powers Property

$$a^m \cdot a^n = a^{m+n}$$

When multiplying the same base, we ADD the exponents and the base stays the same.

Examples

1. $x\left(x^5\right) = x^{1+5} = x^6$
2. $n \cdot m^4 \cdot n^3 \cdot m^2 = m^{4+2}n^{1+3} = m^6 n^4$
3. $4 \cdot 4^2 = 4^{1+2} = 4^3 = 64$
4. $\left(5a^3\right)\left(2a^5\right) = 5 \cdot 2a^{3+5} = 10a^8$

Try These

1. $4^3 \cdot 4^5$
2. $7^5 \cdot 7$
3. $a^4 \cdot a^2$
4. $y^8(y)$
5. $(k^3)(k^6)(k)$
6. $(m^2n)(mn^4)$
7. $(3b^4)(-2b)$
8. $(g^2k^5)(g^7k^3)$
9. $-5x^3(-3x^4)$
10. $(st)(sv)(tu)$
11. $(rt^7)(r^3s)(t^2s^5)$
12. $(8mp^5)(2m^3p^3)(mp)$

Power of a Power Property

$$(a^m)^n = a^{mn}$$

To raise a power to a power, multiply the exponents. If there is a coefficient, don't forget to take that to the power.

Examples

1. $\left(x^2y^4\right)^3 = x^{2\cdot 3}y^{4\cdot 3} = x^6y^{12}$
2. $\left(3w^5\right)^2 = 3^2w^{5\cdot 2} = 9w^{10}$
3. $\left(-2a^3b^6\right)^3 = \left(-2\right)^3 a^{3\cdot 3}b^{6\cdot 3} = -8a^9b^{18}$

Try These

1. $(-x^5)^2$ — 1. ________________
2. $(-x^2)^5$ — 2. ________________
3. $(mn^2)^4$ — 3. ________________
4. $(x^2y^3)^5$ — 4. ________________
5. $(3c^4)^2$ — 5. ________________
6. $(-2a^3)^2$ — 6. ________________
7. $\left(5a^4b^2\right)^3$ — 7. ________________
8. $\left(-4y^5z\right)^3$ — 8. ________________
9. $\left(2d^4e^5\right)^5$ — 9. ________________
10. $\left(-3pqr^2\right)^4$ — 10. ________________

Power of a Product Property

$$(ab)^m = a^m b^m$$

When taking a product to a power, each factor gets raised to the power. If there is a coefficient, don't forget to take that to the power.

Examples

1. $(xy)^3 = x^3 y^3$
2. $(2w)^4 = 2^4 w^4 = 16w^4$
3. $(-3ab)^2 = (-3)^2 a^2 b^2 = 9a^2 b^2$

Try These

1. $(ab)^5$	1. ________________
2. $(xy)^8$	2. ________________
3. $(-2g)^3$	3. ________________
4. $(5xy)^2$	4. ________________
5. $(-3b)^4$	5. ________________
6. $(-mnp)^2$	6. ________________
7. $(6bc)^2$	7. ________________
8. $(-3k)^3$	8. ________________
9. $(9abc)^2$	9. ________________
10. $(-2h)^5$	10. ________________

Quotient of Powers Property

$$\frac{a^m}{a^n} = a^{m-n}$$

When dividing the same bases, subtract the exponents. If there are coefficients, divide them.

Examples

1. $\frac{5^6}{5^4} = 5^{6-4} = 5^2 = 25$
2. $\frac{x^9}{x^7} = x^{9-7} = x^2$
3. $\frac{24m^5n^3}{-6mn^2} = \frac{24}{-6} \cdot m^{5-1}n^{3-2} = -4m^4n$

Try These

1. $\frac{8^7}{8^4}$ — 1. ______________
2. $\frac{3^{15}}{3^{14}}$ — 2. ______________
3. $\frac{d^{12}}{d^7}$ — 3. ______________
4. $\frac{k^8}{k^2}$ — 4. ______________
5. $\frac{x^6y^4}{x^2y^3}$ — 5. ______________
6. $\frac{18m^6n^8}{6mn}$ — 6. ______________
7. $\frac{36a^3b^7c^4}{-9a^2b^5}$ — 7. ______________
8. $\frac{-15p^6q^9}{-5p^3q^7}$ — 8. ______________

Power of a Quotient Property

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

When taking a fraction to a power, take the numerator (top) to the power and the denominator (bottom) to the power.

Examples

1. $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$
2. $\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$
3. $\left(-\frac{n}{3}\right)^3 = \frac{(-n)^3}{3^3} = \frac{-n^3}{27}$

Try These

1. $\left(\frac{5}{2}\right)^2$ 1. ____________________
2. $\left(-\frac{3}{7}\right)^2$ 2. ____________________
3. $\left(-\frac{x}{y}\right)^6$ 3. ____________________
4. $\left(\frac{4}{n}\right)^3$ 4. ____________________
5. $\left(-\frac{3}{b}\right)^3$ 5. ____________________
6. $\left(\frac{m}{5}\right)^3$ 6. ____________________

Unit 8 Objective 6 Remediation

Zero Exponent Property

$$a^0 = 1$$

Anything (except 0) to the zero power is 1.

Examples

1. $8^0 = 1$
2. $x^0 = 1$
3. $2n^0 = 2 \cdot 1 = 2$
4. $-6p^0q^5 = -6 \cdot 1 \cdot q^5 = -6q^5$

Try These

1. $(-5)^0$	1. ____________________
2. m^0	2. ____________________
3. $7g^0$	3. ____________________
4. $-b^0$	4. ____________________
5. $5^0a^5b^0$	5. ____________________
6. $4x^3y^0z$	6. ____________________
7. -9^0x	7. ____________________
8. $-6j^0k^0$	8. ____________________

Negative Exponent Property

$$a^{-m} = \frac{1}{a^m}$$

A negative exponent represents the reciprocal of a number or variable. It does NOT make a number or variable negative.

Examples

1. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
2. $x^{-6} = \frac{1}{x^6}$
3. $m^5 n^{-7} = \frac{m^5}{n^7}$
4. $4x^{-3} = \frac{4}{x^3}$

Try These

1. $(-3)^{-2}$ — 1. ____________________
2. 4^{-3} — 2. ____________________
3. m^{-8} — 3. ____________________
4. $-5b^{-4}$ — 4. ____________________
5. $x^{-2}y^5$ — 5. ____________________
6. $7p^2q^{-6}$ — 6. ____________________
7. $\frac{1}{y^{-5}}$ — 7. ____________________
8. $\frac{j^3}{h^{-4}}$ — 8. ____________________
9. $\frac{1}{6w^{-2}}$ — 9. ____________________

Unit 8 Objective 8 Remediation

Summary of the Exponent Properties	Product of Powers	Power of a Power	Power of a Product
	$a^m \cdot a^n = a^{m+n}$	$\left(a^m\right)^n = a^{m \cdot n}$	$\left(ab\right)^n = a^n b^n$
Quotient of Powers	Power of a Quotient	Zero Exponent	Negative Exponent
$\frac{a^m}{a^n} = a^{m-n}$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$a^0 = 1$	$a^{-n} = \frac{1}{a^n}$

By combining properties, simplify and write each expression with positive exponents only.

1. $\left(-4y^2\right)^3\left(\frac{1}{2}y\right)^2$ 1. ___________________

2. $\left(-mn^2\right)^6$ 2. ___________________

3. $\left(-3xy^2\right)\left(2x^3\right)^3$ 3. ___________________

4. $\left(-5a^3b\right)\left(-ab^2\right)^3$ 4. ___________________

5. $\left(2x^3y^2\right)^4\left(2x^2\right)^2$ 5. ___________________

6. $\left(\frac{a^2}{b}\right)^4$ 6. ___________________

7. $\left(\frac{4f^2}{g^3}\right)^2$ 7. ___________________

8. $\left(\frac{4n^2}{8n^4}\right)^3$ 8. ___________________

9. $\left(\frac{-12gh^2}{4h^3}\right)^2$ 9. ___________________

10. $\left(\frac{xy}{5}\right)^2\left(\frac{10}{xy^2}\right)$

10. ________________

11. $\left(\frac{8p^2q}{24q}\right)^2$

11. ________________

12. $\left(10^3 \cdot 10^{-4}\right)^2$

12. ________________

13. $x^0y^2z^{-3}$

13. ________________

14. $\frac{-15v^5w^3}{25v^7w^0}$

14. ________________

15. $\left(-3a^3b^2\right)^0\left(-7a^5b\right)^2$

15. ________________

16. $\frac{14m^{-3}}{2m}$

16. ________________

17. $\frac{\left(3x^{-2}y^{-1}\right)^3}{18x^3y^2}$

17. ________________

18. $\frac{10\left(-4a^5b\right)^2}{2a^0b^2}$

18. ________________

19. $\frac{5^2\left(5^0\right)^3}{5^{-2}}$

19. ________________

20. $\left(\frac{a^{-2}}{2b^{-3}}\right)^{-3}$

20. ________________

21. $\frac{\left(6q^{-4}\right)\left(q^3\right)^{-2}}{-3q^{-10}}$

21. ________________

Unit 9

Radical Expressions

Objectives:

1. Simplify a radical expression (without variables)
2. Add and/or Subtract radicals (without having to simplify first)
3. Multiply radicals (without variables)
4. Simplify radical expression by rationalizing the denominator

Objectives I must do for Test B: (circle ones you must do)

1 2 3 4

Simplifying Square Roots

Simplifying a square root of a whole number means finding an equivalent expression with the smallest possible number under the radical sign. This is called writing the number in **simplest radical form**.

To write a square root in simplest radical form we first find the **largest possible square factor** of the number under the radical sign. Then we use the Product property.

Product Property of Square Roots

The Product rule states that we can break the square root a number into the square root of product of two factors.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

Example One

Simplify $\sqrt{108}$

Look for the **largest possible perfect square** that will divide evenly into 108. 4, 9 and 36 are perfect squares that divide evenly into 108, but 36 is the largest.

$$\sqrt{108} = \sqrt{36} \cdot \sqrt{3} = 6\sqrt{3}$$

The simplest radical form of $\sqrt{108} = 6\sqrt{3}$.

Example Two

Simplify $\sqrt{192}$

Look for the **largest possible perfect square** that will divide evenly into 192. 4, 16 and 64 are perfect squares that divide evenly into 192, but 64 is the largest.

$$\sqrt{192} = \sqrt{64} \cdot \sqrt{3} = 8\sqrt{3}$$

The simplest radical form of $\sqrt{192} = 8\sqrt{3}$.

Practice

1.) $\sqrt{45}$

2.) $\sqrt{98}$

3.) $\sqrt{125}$

4.) $\sqrt{144}$

5.) $\sqrt{200}$

6.) $\sqrt{72}$

7.) $5\sqrt{18}$

8.) $3\sqrt{28}$

9.) $8\sqrt{128}$

10.) $4\sqrt{80}$

11.) $3\sqrt{27}$

12.) $-3\sqrt{54}$

13.) $-7\sqrt{40}$

14.) $-8\sqrt{121}$

15.) $-4\sqrt{24}$

16.) $3\sqrt{175}$

17.) $5\sqrt{108}$

18.) $2\sqrt{500}$

Addition or Subtraction of Radical Expressions

1. To add or to subtract radicals, the radicals must be exactly the same. The radicand and the index must be exactly the same.
2. The numerical coefficients are then added or subtracted. The radical part stays the same.

Ex. $4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$

Ex. $-4\sqrt{5} + 2\sqrt{5} = -2\sqrt{5}$

Ex. $5\sqrt{7} - 7\sqrt{5} - 2\sqrt{7} = 3\sqrt{7} - 7\sqrt{5}$

Practice

1. $6\sqrt{3} - 8\sqrt{3} =$ __________
2. $-3\sqrt{5} - \sqrt{5} =$ __________
3. $8\sqrt{2} + 4\sqrt{2} =$ __________
4. $2\sqrt{7} - 7\sqrt{7} =$ __________
5. $-2\sqrt{11} + 4\sqrt{11} + \sqrt{12} =$ __________
6. $8 + 2\sqrt{7} - 3\sqrt{6} - 5\sqrt{7} =$ __________
7. $2\sqrt{3} - 7\sqrt{11} + 2\sqrt{11} + 4\sqrt{3} =$ __________
8. $-\sqrt{10} + 8\sqrt{15} + 2\sqrt{15} - 6\sqrt{10} =$ __________
9. $9\sqrt{11} + 5\sqrt{5} - 8\sqrt{11} + 3\sqrt{11} =$ __________
10. $-10\sqrt{2} + 4\sqrt{2} + 8\sqrt{7} - \sqrt{7} =$ __________
11. $7\sqrt{17} + 6\sqrt{15} - 4\sqrt{15} - 8\sqrt{17} =$ __________
12. $-12\sqrt{3} - 4\sqrt{3} + 3\sqrt{5} + 7 =$ __________
13. $\sqrt{24} - 2\sqrt{6} + 4 - \sqrt{49} =$ __________
14. $-2 + 5\sqrt{8} - \sqrt{18} + 9 =$ __________

Multiplying Radicals

1. Multiply the numerical coefficients together and multiply the radicands together.
2. Simplify the new radicand.

Ex. $4\sqrt{3} \cdot 5\sqrt{7} = 4 \cdot 5 \cdot \sqrt{3} \cdot \sqrt{7} = 20\sqrt{21}$

Ex. $2\sqrt{10} \cdot 8\sqrt{2} = 2 \cdot 8 \cdot \sqrt{10} \cdot \sqrt{2} = 16 \cdot \sqrt{20} = 16 \cdot \sqrt{4} \cdot \sqrt{5}$
$= 16 \cdot 2 \cdot \sqrt{5} = 32\sqrt{5}$

Ex. $\left(2\sqrt{5}\right)^2 = \left(2\sqrt{5}\right) \cdot \left(2\sqrt{5}\right) = 4 \cdot \sqrt{25} = 4 \cdot 5 = 20$

Practice

---

1. $\sqrt{2}\sqrt{11} =$ __________

2. $\sqrt{5}\sqrt{7} =$ __________

3. $\sqrt{8}\sqrt{6} =$ __________

4. $3\sqrt{6} \cdot 4\sqrt{14} =$ __________

5. $5\sqrt{10} \cdot 13\sqrt{15} =$ __________

6. $-3\sqrt{5} \cdot 7\sqrt{35} =$ __________

7. $\left(7\sqrt{5}\right)^2 =$ __________

8. $\left(-5\sqrt{11}\right)^2 =$ __________

9. $\left(2\sqrt{17}\right)^2 =$ __________

10. $2\left(\sqrt{100}\right)^2 =$ __________

11. $\sqrt{3}\left(\sqrt{5}+\sqrt{3}\right) =$ __________

12. $\sqrt{6}\left(\sqrt{2}-\sqrt{3}\right) =$ __________

13. $\sqrt{7}\left(\sqrt{2}-\sqrt{14}\right) =$ __________

14. $\sqrt{11}\left(\sqrt{22}+\sqrt{33}\right) =$ __________

15. $\sqrt{5}\left(\sqrt{8}+\sqrt{12}\right) =$ __________

16. $\sqrt{8}\left(\sqrt{2}-\sqrt{5}\right) =$ __________

17. $\sqrt{2}\left(\sqrt{6}+\sqrt{8}\right) =$ __________

18. $\sqrt{13}\left(\sqrt{2}-\sqrt{8}\right) =$ __________

Simplify Radical Expressions by Rationalizing the Denominator:

1. If there is more than one radical sign, put the numerator and denominator under one radical sign and reduce the fraction, if possible. If there is one radical sign, reduce the fraction, if possible.
2. Split the radical into two separate radicals.
3. Look at the denominator. What does the denominator need to be multiplied by to create a perfect square in the denominator?
4. Multiply the numerator and denominator by this term as a radical.
5. Simplify the numerator and the denominator by pulling out the perfect square factor.
6. Double check to see if the fraction remaining can be reduced. Remember the values under the radical sign can be reduced together and the values in front of the radical sign can be reduced together.

Ex. $\sqrt{\frac{75}{3}} = \sqrt{25} = 5$

Ex. $\frac{\sqrt{16}}{\sqrt{4}} = \sqrt{\frac{16}{4}} = \sqrt{4} = 2$

Ex. $\frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{9}} = \frac{4\sqrt{3}}{3}$

Ex. $\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{\sqrt{25}} = \frac{\sqrt{15}}{5}$

Note that the last two examples CANNOT be reduced because one factor is under the radical sign and one factor is not under the radical sign.

Practice

1. $\frac{\sqrt{24}}{\sqrt{6}}$ = __________

2. $\sqrt{\frac{15}{3}}$ = __________

3. $\frac{\sqrt{13}}{\sqrt{2}}$ = __________

4. $\frac{\sqrt{7}}{\sqrt{3}}$ = __________

5. $\sqrt{\frac{1}{5}}$ = __________

6. $\frac{4}{\sqrt{7}}$ = __________

7. $\sqrt{\frac{128}{2}}$ = __________

8. $\frac{\sqrt{18}}{\sqrt{10}}$ = __________

9. $\frac{5}{\sqrt{10}}$ = __________

10. $\frac{\sqrt{28}}{\sqrt{6}}$ = __________

11. $\sqrt{\frac{9}{10}}$ = __________

12. $\frac{7}{\sqrt{3}}$ = __________

Unit 10

Operations with Polynomials

Objectives:

1. Classify a polynomial by degree and/or number of terms
2. Add polynomial expressions and express answer in simplest form
3. Subtract polynomial expressions and express answer in simplest form
4. Multiply polynomial expression by a monomial and express answer in simplest form
5. Multiply two binomials and express answer in simplest form
6. Multiply binomial by a trinomial and express answer in simplest form
7. Use multiple operations to simplify a polynomial
8. Find the GCF of monomials
9. Find the LCM of monomials

Objectives I must do for Test B: (circle ones you must do)

1 2 3 4 5 6 7 8 9

Unit 10 Objective 1 Remediation

Naming a Polynomial

Highest Degree		Number of Terms	
Constant	0	Monomial	1
Linear	1	Binomial	2
Quadratic	2	Trinomial	3
Cubic	3	Polynomial	4 or more
n^{th} degree	n		

Name each polynomial by degree and number of terms.

1. $-m^5$ ________________ ________________

2. $2x^3 + x^2 - 7$ ________________ ________________

3. $4 - 9y^2$ ________________ ________________

4. $v + w + x + y + z$ ________________ ________________

5. $3a^7 - 9$ ________________ ________________

6. $6x^4 - 3x^2 + x$ ________________ ________________

7. -8 ________________ ________________

8. $m + n$ ________________ ________________

9. $p^6 - p^3 - p^2 - p$ ________________ ________________

10. $4x^2 - 2xy + y^2$ ________________ ________________

11. $7h^3 - j^2$ ________________ ________________

12. $\frac{1}{2}b^2$ ________________ ________________

Unit 10 Objective 2 Remediation

Adding Polynomials

Adding polynomials is **combining like terms**. Like terms are terms that have the **same variable part and same exponent.**

Example One

Simplify $(4x - 8y + 1) + (-x + 5y - 9)$

Because we are adding polynomials, we can drop the parentheses and combine the like terms

$$(4x + -x) + (-8y + 5y) + (1 + -9)$$

Answer: $\boldsymbol{3x - 3y - 8}$

Example Two

Simplify $(-2x^2 + 3x - 5) + (3x^2 + 5x - 2)$

Because we are adding polynomials, we can drop the parentheses and combine the like terms

$$(-2x^2 + 3x^2) + (3x + 5x) + (-5 - 2)$$

Answer: $\boldsymbol{x^2 + 8x - 7}$

Example Three

Simplify $(6a^2 - ab - 2b^2) + (a^2 + 3ab + 2b^2)$

Because we are adding polynomials, we can drop the parentheses and combine the like terms

$$(6a^2 + a^2) + (-ab + 3ab) + (-2b^2 + 2b^2)$$

Answer: $\boldsymbol{7a^2 + 2ab}$

Try These

1. $(5a^2 - 2) + (12a^2 + 5)$

2. $(-3a^2 - a + 8) + (7a^2 + 7a - 1)$

3. $(2a^2 - 11a - 4) + (-9a^2 - 2a - 15)$

4. $(8x + x^2 + 6) + (4 + x - 3x^2)$

5. $(3x^3 + 2x - 3) + (8x^3 - 5x - 10)$

6. $(y^3 - 6y^2 - 7) + (-4y^3 - y + 7)$

7. $(z^3 - 6z^2 - 7) + (-4z^3 - z + 7)$

8. $(2x^4 + 5x^2 - 13) + (-7x^4 - 8x^2 + 1)$

9. $(-5x^4 + 2x^3 - 8x^2 - x) + (x^3 + 8x^2 - 3x - 1)$

10. $(7y^4 - 4y^2 + 12) + (-8y^4 + 3y^3 + 4y^2 + y)$

11. $(3x^2 - 2xy + 9y^2) + (x^2 - 5xy - 6y^2)$

12. $(-12x^2 + xy + 2y^2) + (-4x^2 + 9xy - 2y^2)$

Subtracting Polynomials

To subtract polynomials, you must distribute the subtraction sign to all of the terms in the parenthesis and change the subtraction sign into an addition sign. Next, finish the problem as an addition problem by **combining like terms**. Like terms are terms that have the **same variable part and same exponent.**

Example One

Simplify $(4x - 8y + 1) - (-x + 5y - 9)$

Distribute the subtraction sign to all of the terms in the parenthesis and change the subtraction sign into an addition sign.

$$(4x - 8y + 1) - (-x + 5y - 9)$$

$$(4x - 8y + 1) + (+x - 5y + 9)$$

$$(4x + x) + (-8y - 5y) + (1 + 9)$$

Answer: $\mathbf{5x - 13y + 10}$

Example Two

Simplify $(-2x^2 + 3x - 5) - (3x^2 + 5x - 2)$

Distribute the subtraction sign to all of the terms in the parenthesis and change the subtraction sign into an addition sign.

$$(-2x^2 + 3x - 5) - (3x^2 + 5x - 2)$$

$$(-2x^2 + 3x - 5) + (-3x^2 - 5x + 2)$$

$$(-2x^2 - 3x^2) + (3x - 5x) + (-5 + 2)$$

Answer: $\mathbf{-5x^2 - 2x - 3}$

Try These

1. $(7x + 4) - (2x + 9)$

2. $(3x + 12) - (5x - 6)$

3. $(-4x^2 + 10) - (6x^2 - 9)$

4. $(2x^2 + 3x + 8) - (x^2 + 5x - 1)$

5. $(-x^2 + 9x - 2) - (9x^2 - 4x + 4)$

6. $(3x^2 + 7x + 1) - (8 + 5x + x^2)$

7. $(4x^3 + 6x^2 - 8x) - (x^3 - 2x^2 + 12x)$

8. $(x^3 + 2x^2 + 5x) - (3x^2 - x - 7)$

9. $(x^4 + 8x^2 - 1) - (x^2 - 3x^3 + x^4)$

10. $(5x^4 - 2x^2) - (3x - 2x^2 - 4x^3 + 6x^4)$

11. $(3x^2 + 7xy - 2y^2) - (x^2 - 6xy + 2y^2)$

12. $(-x^2 - 9xy + 5y^2) - (4x^2 - 2xy - y^2)$

13. $(4x^2y - 3xy^2) - (3x^2y - 8xy^2)$

14. $(8x^2 + 3xy - 20y^2) - (-4x^2 + 3xy + 20y^2)$

Multiplying a Monomial and a Polynomial

Example One

Simplify $-3(2x+3y-2)$

Distribute the term in the front of the parenthesis to all of the terms in the parenthesis.

$$(-3)2x+(-3)3y-(-3)2$$

Multiply each term by the term in the front.

Answer: $\boldsymbol{-6x-9y+6}$

Example Two

Simplify $2x^2(-2x^2+3x-5)$

Distribute the term in the front of the parenthesis to all of the terms in the parenthesis.

$$(2x^2)(-2x^2)+(2x^2)3x-(2x^2)5$$

Multiply each term by the term in the front. Multiply the coefficients. Also, remember when you multiply the same bases, you add the powers.

Answer: $\boldsymbol{-4x^4+6x^3-10x^2}$

Example Three

Simplify $3ab^2(6a^2-ab+7b^2)$

Distribute the term in the front of the parenthesis to all of the terms in the parenthesis.

$$(3ab^2)6a^2-(3ab^2)ab+(3ab^2)7b^2$$

Multiply each term by the term in the front. Multiply the coefficients. Also, remember when you multiply the same bases, you add the powers.

Answer: $\boldsymbol{18a^3b^2-3a^2b^3+21ab^4}$

Try These

1. $7(m^2+5)$

2. $-3(8m^2-4m)$

3. $2m(m^3+9)$

4. $m^2(-5m-6)$

5. $9(4a^2-a+2)$

6. $3a(12+5a-a^2)$

7. $-4a^2(7a^2+15a-1)$

8. $2a^3(6a^2-2a+3)$

9. $x^2y(x^2-y^2)$

10. $-5xy^2(-x^3y+4xy^3)$

11. $9xy(2x^2y+9xy-4xy^2)$

12. $-x^2y^2(5x^2-8xy+y^2)$

13. $3cd^4(2c^4-5c^2d^2-18d^4)$

14. $8c^2d^2(3c^4d^3+10c^3d^4+11)$

15. $-9c^7d^3(16c^5d^2-5c^2d^5)$

16. $4c^5(3c^2-20cd-3d^2)$

Multiplying Two Binomials

When multiplying two binomials we use **F O I L** to remember the steps.

F – Multiply the first terms in each parenthesis
O – Multiply the outside terms
I – Multiply the inside terms
These are usually like terms and get added
L – Multiply the last terms

Example One

Multiply: $(x+5)(x-3)$

F O I L

$$x^2 - 3x + 5x - 15$$

Add the **OI**

Answer: $\boldsymbol{x^2 + 2x - 15}$

Example Two

Multiply: $(2x-3)(5x-1)$

F O I L

$$10x^2 - 2x - 15x + 3$$

Add the **OI**

Answer: $\boldsymbol{10x^2 - 17x + 3}$

Example Three

Multiply: $(3x+5y)(x+4y)$

F O I L

$$3x^2 + 12xy + 5xy + 20y^2$$

Add the **OI**

Answer: $\boldsymbol{3x^2 + 17xy + 20y^2}$

Try These

1. $(x+4)(x+2)$

2. $(x+7)(x+1)$

3. $(x-6)(x-3)$

4. $(x+8)(x-2)$

5. $(x-7)(x+4)$

6. $(x-9)(x-2)$

7. $(2x+4)(x+1)$

8. $(3x+7)(x-3)$

9. $(4x-2)(5x-1)$

10. $(7x-4)(3x+2)$

11. $(5x-8)(4x-4)$

12. $(2x+y)(x+3y)$

13. $(2x+1)(9x-5)$

14. $(3x-y)(8x-y)$

15. $(2x+y)(4x-3y)$

16. $(5x-2y)(3x+4y)$

17. $(7x+3y)(x+2y)$

18. $(6x+6y)(2x-4y)$

Multiplying a Binomial and a Trinomial

Remember the distributive property? We use this to multiply polynomials. Let's take a look at an example…

Multiply $(x+3)(2x^2-3x+2)$

Take each term of the first polynomial and distribute it to each of the terms in the second polynomial.

First, distribute the x. Multiply the x times all of the terms in the second polynomial.

$$(x+3)(2x^2-3x+2)=2x^3-3x^2+2x$$

Next, distribute the + 3. Multiply the 3 times all of the terms in the second polynomial.

$$(x+3)(2x^2-3x+2)=2x^3-3x^2+2x+6x^2-9x+6$$

Combine the like terms.

$$(x+3)(2x^2-3x+2)=2x^3\underline{-3x^2}+2x+\underline{6x^2}-9x+6$$

So our answer is $(x+3)(2x^2-3x+2)=\boxed{2x^3+3x^2-7x+6}$

Try This

1. Multiply $(x+1)(x^2-2x-5)$

2. Multiply $(x^2+3x-8)(2x+5)$

3. Multiply $(3x-1)(x^2-7x+3)$

4. Multiply $(3x^2-4x-2)(2x-3)$

5. Multiply $(x+2)(x^2-5x+4)$

6. Multiply $(2x+5)(x^2-4x+3)$

7. Multiply $(5x-1)(-2x^2+4x-2)$

8. Multiply $(3x+10)(4-x+6x^2)$

9. Multiply $(2x^2-3)(4x^3-x^2+7)$

Use Multiple Operations to Simplify a Polynomial

To do these problems, use the order of operations. The pneumonic PEMDAS will help you remember what to do.

P – Parentheses are always done first.
E – Exponents are done second.
MD – Multiplication and Division are done third. Do these operations as they are seen in the problem, reading left to right.
AS – Addition and Subtraction are done last. Do these operations as they are seen in the problem, reading from left to right.

Ex $2x(x-4)+4x(2x^2-3)=2x^2-8x+8x^3-12x$ **Eliminate Parentheses.**

$=8x^3+2x^2-20x$ **Add like terms.**

Ex $3-8(3x+4)=3-24x-32$ **Eliminate Parentheses.**

$=-24x-29$ **Add like terms.**

Ex $2(4x-3x^2)-5x(-2x^2+3x-1)=8x-6x^2+10x^3-15x^2+5x$

Eliminate Parentheses.

$=10x^3-21x^2+13x$ **Add like terms.**

Use the order of operations to simplify each expression.

1. $-3(2x+4)+2(-5x-3)$

2. $6-2(5x+10)$

3. $4x(x^2-5x+8)+8x-12$

4. $3x-9x(4x-8)$

5. $2(3x-4)+3(3x^2-6x+1)$

6. $9x-(7x^2-3x+8)$

7. $5(2x^2-6x-10)-4(x^2+8x+3)+3(2x^2-8x+2)$

8. $5x^3-7x(7x+10)-2x^2(9x-11)$

9. $x^2-4x^2(3x+9)+6x$

10. $-3(x^2-4x+7)-7(2x^2+6)$

11. $-(6x^3-4x-3)+3x-4(5x^2-8x+2)$

12. $2x(x+3)-5x(3-2x)+8x(4x-1)$

13. $9-2(x-7)^2+8$

14. $4x(3x-5x^3-6)+7(2x^2-4x)$

15. $3x-(8x-2)(3x+1)$

Unit 10 Objective 8 Remediation

Find the GCF of Monomials

Example:

Find the GCF of $15x^2yz$ and $24x^3y^2$

Step 1: Find the GCF of the coefficients by listing the factors of each number

15: 1, 3, 5, 15

24: 1, 2, 3, 4, 6, 8, 12, 24

The greatest factor that is in both 15 *and* 24 is 3.

Step 2: Find the GCF of the variables

- Look to see which variable(s) are in ALL of the terms.
  - In our example x and y are in both terms
  - The variable z is not in one of the terms so it is not in the GCF

- Find the smallest exponent of each variable
  - The smallest exponent for x is x^2
  - The smallest exponent for y is y

The GCF of the variables is x^2y

Step 3: Combine for GCF

The GCF of the coefficients was 3 and the GCF of the variables is x^2y

Thus, the GCF of $15x^2yz$ and $24x^3y^2$ is: $\mathbf{3x^2y}$

Try Some:

Find the GCF of each set of monomials.

1.) $20x^3y$; $35xy^2$

2.) $7a^3b^4$; $21a^2b^2$

3.) $12x^3y^2$; $36x^2$

4.) $42a^4b^6$; $28ab^4$

5.) $10xy^3z^2$; $21x^2y^2z$

6.) $a^2b^4c^3$; $8ac^6$

7.) $16x^2$; $28x^5$; $32x^3$

8.) $40a^2b$; $16a^3b^3$; $32a^2$

9.) $21x^4z^2$; $24x^3y^2$

10.) $63a^3bc^3$; $36a^2b^2c$; $45ab^3c^4$

Find the LCM of Monomials

LCM stands for Least Common Multiple. We are looking for the smallest multiple that the monomials have in common. A multiple of a number is what you get when you multiply your number by 1, 2, 3, etc. For example, the multiples of 5 are: 5, 10, 15, 20, 25, etc.

Example:

Find the LCM of $15x^2yz$ and $20x^3y^2$

Step 1: Find the LCM of the coefficients by listing the factors of each number

15: 15, 30, 45, **60**, 75, 90, 105, …

20: 20, 40, **60**, 80, 100, …

The smallest multiple of both 15 *and* 20 is 60.

Step 2: Find the LCM of the variables

- If the variable shows up in any term it is part of the LCM
  - In our example x, y, and z all show up in at least one of the terms
  - This means that x, y, and z are all part of the LCM.

- Find the largest exponent of each variable
  - The largest exponent for x is x^3
  - The largest exponent for y is y^2
  - The largest exponent for z is just z.

The LCM of the variables is x^3y^2z

Step 3: Combine for LCM

The LCM of the coefficients was 60 and the LCM of the variables is x^3y^2z

Thus, the LCM of $15x^2yz$ and $20x^3y^2$ is: $\boldsymbol{60x^3y^2z}$

Try Some:

Find the LCM of each set of monomials.

1.) $20x^3y$; $35xy^2$

2.) $7a^3b^4$; $21a^2b^2$

3.) $12x^3y^2$; $36x^2$

4.) $42a^4b^6$; $28ab^4c$

5.) $10xy^3z^2$; $21x^2y^2z$

6.) $a^2b^4c^3$; $8ac^6$

7.) $16x^2$; $28x^5yz$

8.) $40a^2b$; $16a^3b^3$

9.) $21x^4z^2$; $24x^3y^2$

10.) $36a^2b^2c$; $45ab^3$

Unit 11

Factoring Polynomials

Objectives:

1. Factor a polynomial using the GCF
2. Factor a difference of squares
3. Factor a polynomial in the form $x^2 + bx + c$
4. Factor a polynomial in one step using the different factoring methods from objectives 2-5
5. Factor a polynomial in two steps where the first step is factoring out the GCF and the second step is factoring a difference of squares or $x^2 + bx + c$
6. Simplify a rational algebraic expression

Objectives I must do for Test B: (circle ones you must do)

1 2 3 4 5 6

Unit 11 Objective 1 Remediation

Factoring using GCF

Example:

Factor $3x^3 + 27x^2 + 9x$

1.) To factor out the GCF in an expression like the one above, first find the GCF of all of the expression's terms (like objective 1 in this unit).

The GCF of $3x^3, 27x^2, \text{and } 9x$ is: $3x$

2.) Next, write the GCF on the left of a set of parentheses:

$3x(\qquad)$

3.) Next, divide each term from the original expression ($3x^3 + 27x^2 + 9x$) by the GCF ($3x$), then write it in the parenthesis.

$(3x^3) \div (3x) = x^2$ $\qquad$ $(27x^2) \div (3x) = 9x$ $\qquad$ $(9x) \div (3x) = 3$

Answer: $3x(x^2 + 9x + 3)$

4.) Check your answer by using the distributive property and multiply each term inside the parentheses by 3x:

$$3x(x^2 + 9x + 3) = 3x^3 + 27x^2 + 9x$$

If you factor $\mathbf{3x^3 + 27x^2 + 9x}$ your final answer will be $\mathbf{3x(x^2 + 9x + 3)}$

Try Some:

Factor each polynomial using the GCF.

1.) $21a^3 - 14a^2$

2.) $4x^3 + 32x$

3.) $10a - 35b + 15$

4.) $21c^3 - 14c$

5.) $3a^3 + 6a^2 - 12a$

6.) $10x^3 - 5x^2 + 20x$

7.) $5y^3 - 10y^2 + 15y$

8.) $18x^3 - 6x^2 + 24x$

9.) $8ab^2 - 12a^2b$

10.) $3a^2b^2 + 18ab$

11.) $6xy^3 - 24xy^2 - 12xy$

12.) $20x^2y^4 + 35x^3y^3 + 15x^4y^2$

Factoring a Difference of Squares

Example:

Factor $4x^2 - 25$

Step 1: The first step at factoring this is to make sure that the expression is a difference between squares. Ask yourself the following questions:

Question	Answer and Reason
Are there only two terms?	Yes, $4x^2$ and 25
Are both terms ($4x^2$ and 25) perfect squares?	Yes, $4x^2$ and 25 are both perfect squares (($(2x)^2 = 4x^2$ and $5^2 = 25$)
Is the 2nd term being subtracted from the first?	Yes, $4x^2 - 25$

Since we answered YES to all 3 questions, we know it is a difference of squares and can write out our 2 sets of parentheses, one with a plus sign and the other with a minus sign:

$$(\quad + \quad)(\quad - \quad)$$

Step 2: Now find the square root of $4x^2$ (the first term). The square root of the entire term is 2x since $2^2 = 4$ and $x \cdot x = x^2$. Write this term on the left inside of each set of parentheses.

$$(2x + \quad)(2x - \quad)$$

We will now consider 25. Find the square root of 25, which is 5. So 5 is written on the right inside of each set of parentheses.

$$(2x + 5)(2x - 5)$$

If you factor $4x^2 - 25$your final answer will be $(\mathbf{2x+5})(\mathbf{2x-5})$

Try Some:

Factor each polynomial.

1.) $b^2 - 16$

2.) $f^2 - 81$

3.) $36 - x^2$

4.) $9x^2 - 16$

5.) $49n^2 - 1$

6.) $4a^2 - 9$

7.) $a^4 - 36$

8.) $49a^2 - 25b^2$

9.) $100 - 121x^2$

10.) $x^2 - 64y^2$

11.) $a^2 + 100$

12.) $64 + y^2$

Unit 11 Objective 3 Remediation

Factoring a Trinomial in the form $x^2 + bx + c$

If c is Positive	If c is Negative
Find 2 numbers that: Multiply to c Add up to b Their signs will be the same	Find 2 numbers that: Multiply to c Have difference of b Their signs will be different.

Examples:

1.) Factor $x^2 + 9x + 18$

c is positive (+15) so we are looking for 2 numbers that *multiply to 18* and *add up to 9*. We also know that the numbers will have the *same sign*. Since the b value is positive (9) both signs will be addition. So we can fill in our parentheses as follows: $(x + \quad)(x + \quad)$

Numbers that multiply to 18:

1, 18 → add up to 19
2, 9 → add up to 11
3, 6 → add up to 9

Since 3 and 6 multiply to 18 and add up to 9, these are the number we need to factor $x^2 + 9x + 18$, so we can fill them in our parentheses that we started above: $(x + 3)(x + 6)$

If you factor $x^2 + 9x + 18$ your final answer will be $(x + 3)(x + 6)$

2.) Factor $x^2 - 4x - 12$

c is negative (-12) so we are looking for 2 number that *multiply to 12* and have a *difference of 4*. We also know that the numbers will have *different signs*. So we can fill in our parentheses as follows: $(x + \quad)(x - \quad)$

Numbers that multiply to 12:

1, 12 → have difference of 11
2, 6 → have difference of 4
3, 4 → have difference of 1

Since 2 and 6 multiply to 12 and have difference of 4, these are the numbers we need to factor $x^2 - 4x - 12$. Since b is negative, the larger number (6) will have the negative sign. So we can fill in our parentheses as follows: $(x + 2)(x - 6)$

If you factor $x^2 - 4x - 12$ your final answer will be $(x + 2)(x - 6)$

Try Some:

1.) $x^2 + 4x + 3$

2.) $a^2 - 2a - 3$

3.) $c^2 - 9c + 14$

4.) $b^2 + 3b - 10$

5.) $z^2 - z - 56$

6.) $n^2 + 18n + 30$

7.) $a^2 - 3a - 40$

8.) $x^2 - 14x + 45$

9.) $w^2 - 19wv - 20v^2$

10.) $a^2 + 7ab + 6b^2$

Factoring a Polynomial in One Step Using either GCF, Difference of Squares or Trinomial Factoring in the form $x^2 + bx + c$

Look for a GCF First

How many terms are in the polynomial?

2 Terms

Is it a Difference of Squares?

$x^2 - y^2 = (x + y)(x - y)$

3 Terms

$x^2 + bx + c$

Find two numbers that multiply to c and add to b

Example One

Factor $x^2 - x - 56$

- ✧ Look for a GCF FIRST. Does the polynomial have a GCF? **No**
- ✧ How many terms does the polynomial have? **3 terms**
- ✧ Use trinomial factoring. Find two numbers that multiply to -56 and add to -1.
  The numbers are -8 and 7.

Answer: $(x - 8)(x + 7)$

Example Two

Factor $9x^2 - 18x$

- ✧ Look for a GCF FIRST. Does the polynomial have a GCF? **Yes**
- ✧ Determine the GCF and factor it out. The GCF is $9x$.

Answer: $9x(x - 2)$

Example Three

Factor $4x^2 - 81$

- ✧ Look for a GCF FIRST. Does the polynomial have a GCF? **No**
- ✧ How many terms does the polynomial have? **2 terms**
- ✧ Is it a difference of squares? **Yes**

Answer: $(2x + 9)(2x - 9)$

Factor the following.

1. $x^2 - 9$
2. $144x^4 - y^2$
3. $x^2 + 4x - 12$
4. $x^2 + 9x + 8$
5. $x^2 - 81y^2$
6. $x^2 - 17x + 72$
7. $-28x^3y^2 + 7x^2y^2 - 35x^2y^3$
8. $6x^2 + 24x$
9. $x^2 - 5xy - 36y^2$
10. $24x^4 - 8x^3$
11. $16 - 25x^2$
12. $15x^4y - 10x^3y + 5x^2y$
13. $x^2 + 6x + 9$
14. $x^2 - 11xy + 30y^2$
15. $49 - x^2$
16. $12x^3 + 144x^2 - 36x$
17. $x^2 - 8x - 84$
18. $169x^2 - 16y^2$
19. $x^2y^2 - x^3y^3$
20. $x^2 - 12x + 36$

Factoring a Polynomial in Two Steps, First by GCF and Second as a Difference of Squares or Trinomial Factoring in the form $x^2 + bx + c$

Look for GCF FIRST
Factor it out

How many terms are in the polynomial?

2 Terms

Is it a Difference of Squares?

$x^2 - y^2 = (x + y)(x - y)$

3 Terms

$x^2 + bx + c$

Find two numbers that multiply to c and add to b

Example One

Factor $27x^3 - 12x$

- ✧ Look for a GCF FIRST. Does the polynomial have a GCF? **Yes**
- ✧ Determine the GCF and factor it out. The GCF is $3x$.

$$3x(9x^2 - 4)$$

- ✧ Look inside the parentheses and determine the number of terms. **2 terms**
- ✧ Is it a difference of squares? **Yes**

Answer: $3x(3x + 2)(3x - 2)$

Example Two

Factor $2x^4y + 16x^3y + 32x^2y$

- ✧ Look for a GCF FIRST. Does the polynomial have a GCF? **Yes**
- ✧ Determine the GCF and factor it out. The GCF is $2x^2y$.

$$2x^2y(x^2 + 8x + 16)$$

- ✧ Look inside the parentheses and determine the number of terms. **3 terms**
- ✧ Use trinomial factoring. Find two numbers that multiply to 16 and add to 8. The numbers are 4 and 4.

Answer: $2x^2y(x + 4)^2$

Factor the following.

1. $2x^2 - 8xy + 8y^2$
2. $5x^2 - 20$
3. $x^3 + 6x^2 - 27x$
4. $x^3y - 100xy$
5. $3x^4 - 3x^3 - 90x^2$
6. $36y^4 - 49y^2x^2$
7. $4x^2 - 64x + 256$
8. $x^4y^2 - 25x^2y^2$
9. $2x^3 - 6x^2 - 56x$
10. $5x^2 - 20x^4$
11. $x^2y^3 + 7xy^3 - 98y^3$
12. $8x^3y^4 - 72xy^2$
13. $6x^2 + 36xy + 30y^2$
14. $3x^2y - 36xy^2 + 81y^3$
15. $81x^2 - 9y^2$
16. $2x^3y + 14x^2y - 240xy$

Simplifying Rational Expressions

Notes

- A rational expression is an expression in the form $\frac{P}{Q}$, where P and Q are polynomials and $Q \neq 0$.
- A rational expression is in simplest form when the numerator and denominator have no common factors other than $1 \text{ or } -1$.
- Be sure to completely factor both numerator and denominator

Examples

a.) Simplify $\frac{6x^2-9x}{12x^2}$

- First factor the numerator and denominator: $\frac{3x(2x-3)}{12x^2}$
- Cross out common factors: $\frac{3x(2x-3)}{12x^2} \rightarrow \frac{\cancel{3x}(2x-3)}{\underbrace{\cancel{12x^2}}_{4x}} \rightarrow \frac{2x-3}{4x}$
- Thus, the simplified form is: $\frac{2x-3}{4x}$

b.) Simplify $\frac{x^2+5x+6}{x^2+6x+8}$

- First, completely factor the numerator and denominator: $\frac{x^2+5x+6}{x^2+6x+8} = \frac{(x+3)(x+2)}{(x+2)(x+4)}$
- Cross out common factors: $\frac{(x+3)\cancel{(x+2)}}{\cancel{(x+2)}(x+4)} \rightarrow \frac{(x+3)}{(x+4)}$
- Thus, the simplified form is: $\frac{(x+3)}{(x+4)}$

Try Some

1.) $\frac{14t}{7t-7}$

2.) $\frac{3m+9}{6m-12}$

3.) $\frac{4m^2}{8m+12m^2}$

4.) $\frac{3y+6}{y^2+3y+2}$

5.) $\frac{x^2-1}{x^2+2x+1}$

6.) $\frac{x^2-6x+9}{x^2-7x+12}$

7.) $\frac{2x^2+x-1}{2x^2+11x-6}$

8.) $\frac{x^2-1}{x^2+11x+10}$