Keystone Review
Module 2

The Keystone Exam is broken into 2 modules.

Module 1	Module 2
• Operations with Real Number and Expressions • Linear Equations • Linear Inequalities	• Functions • Slope, Intercepts, Rate of Change • Writing Equations • Data Analysis

Functions

Relation – a pairing between 2 sets of numbers

Function – a relation in which each input has exactly 1 output; x's don't repeat

Domain – Input values; x-values

Range – Output values; y-values

Determining if a Relation is a Function

Determine if the relation is a function.

1.) {(–1, 0), (–1, 1), (–4, 4), (–2, –4)}

no; x = -1 repeats

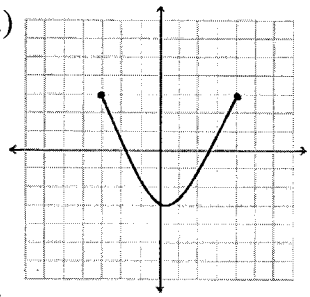
2.)

x	y
–2	3
–5	5
6	7
8	–2

yes it is a function

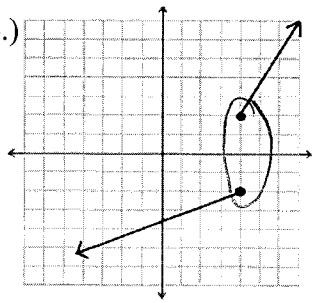
#3-5 use vertical line test

3.)



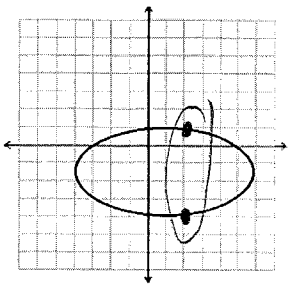
yes

4.)



no

5.)



no

vertical line test: if any vertical line hits the graph at 2 or more spots then it is not a function

Try Some

6.) Which relation is also a function?

a.) {(0, 3), (4, 3), (8,3)}

b.) {(1, 2), (2, 3), (1, 4)}

c.) {(4, 5), (4, 7), (4, 9)}

d.) {(6, 0), (8, 1), (8, 5)}

7.) Which table of values represents a function?

a.)

x	y
10	-5
-2	-5
8	-1
10	-9

b.)

x	y
6	9
6	6
6	3
6	0

c.)

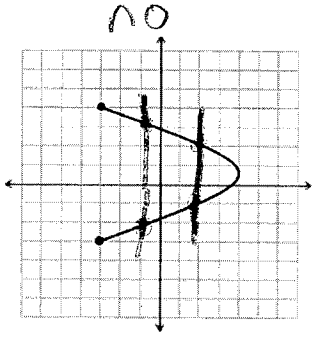
x	y
1	-5
4	-2
-1	8
0	0

d.)

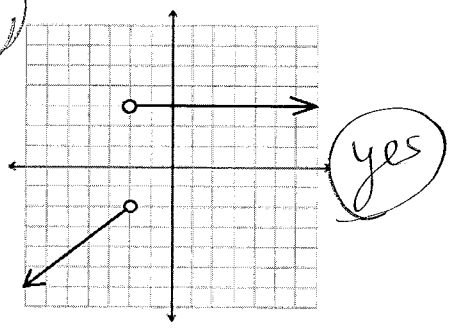
x	y
2	0
-2	1
0	2
-2	-2

8.) Which graph represents a function?

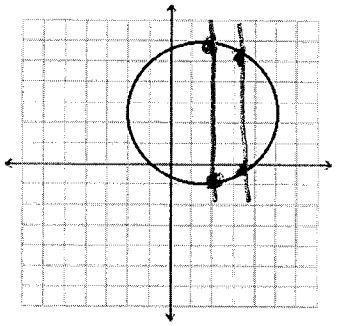
a.)



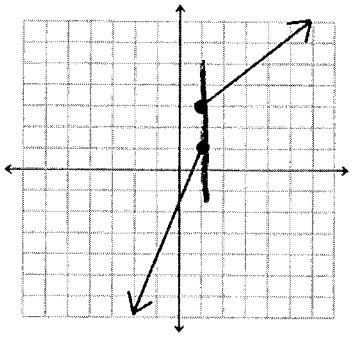
b.)



c.)



d.)



Domain of a graph is the pt. farthest left to the point farthest right

Range of a graph is lowest pt. to the highest pt.

Finding Domain and Range

x-values ↓ y-values ↓

What is the domain and range of the following relations?

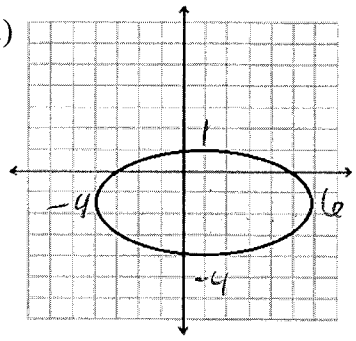
1.) $\{(1,4),(3,5),(5,4),(8,6),(0,4)\}$

D: $\{0, 1, 3, 5, 8\}$

R: $\{4, 5, 6\}$

with graphs when you see an arrow it should be $+\infty$ or $-\infty$

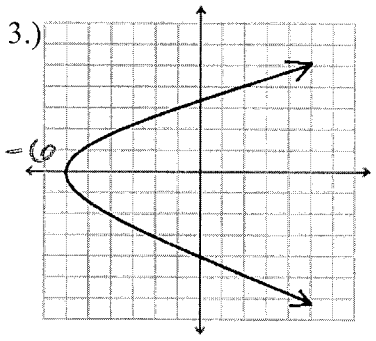
2.)



D: $[-4, 6]$

R: $[-4, 1]$

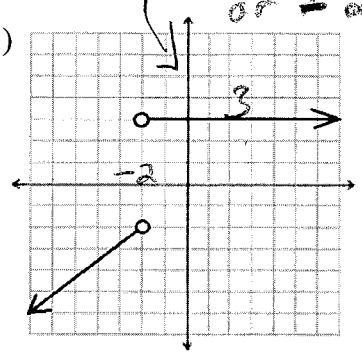
3.)



D: $[6, \infty)$

R: $(-\infty, \infty)$

4.)



D: $(-\infty, -2)(-2, \infty)$

R: $(-\infty, 3]$

Try Some

Determine the domain and range of each relation.

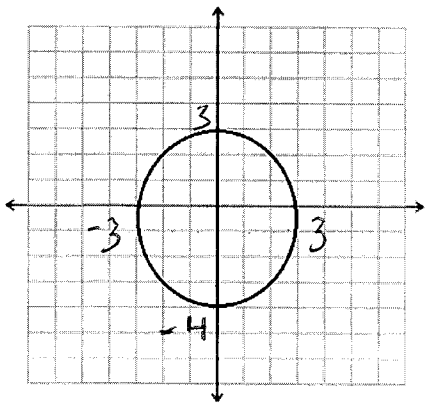
5.)

x	12	10	8	8	8
y	6	7	8	9	10

D: $\{8, 10, 12\}$

R: $\{6, 7, 8, 9, 10\}$

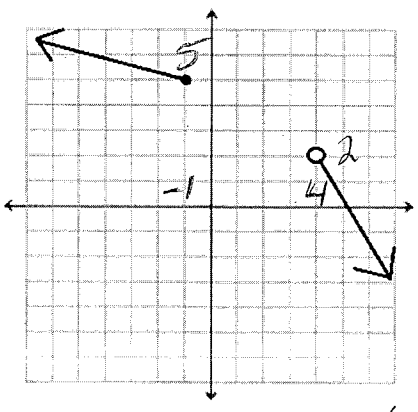
6.)



D: $[-3, 3]$

R: $[-4, 3]$

7.)



D: $(-\infty, -1]\ (4, \infty)$

R: $(-\infty, 2)\ [5, \infty)$

Slope, Intercepts, Writing Equations from Graphs, Rate of Change

Slope – Steepness of a line; $\frac{rise}{run}$ **Slope Formula:** $m = \frac{y_2 - y_1}{x_2 - x_1}$

positive Slope

x-intercept – point where the graph crosses the x-axis (x-value when $y = 0$)
y-intercept – point there graph crosses the y-axis (y-value when $x = 0$)

negative Slope

To write the equation of the line:

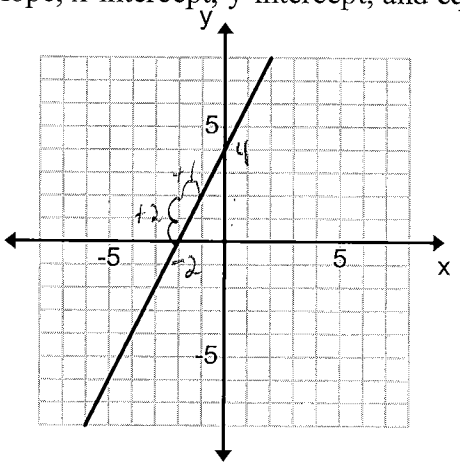
Slope-Intercept Form: $y = mx + b$ (slope, y-intercept)

Slope is undefined

Slope=0

Finding Slope and Intercepts; Writing an Equation from a Graph

1.) Identify the slope, x-intercept, y-intercept, and equation of the graph.



Slope: $\frac{2}{1}$

x-intercept: $(-2, 0)$

y-intercept: $(0, 4)$

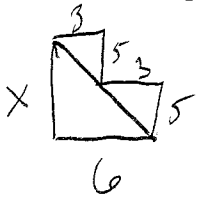
Equation of Line: $y = 2x + 4$

2.) What is the slope of the line that passes through $(-3, 8)$ and $(2, -2)$?

(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 8}{2 + 3} = \frac{-10}{5} = \boxed{-2}$$

3.) A pole is placed against a house, 6 feet from the base of the wall. In this position, the pole has a slope of $\frac{5}{3}$. What height off the ground does the top of the pole rest against the house?



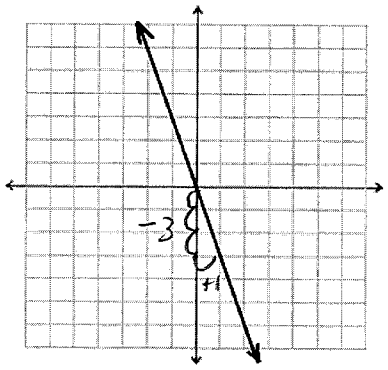
10 feet

rise / run

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Try Some

3.) Identify slope and intercepts of the graph:



Slope: $\frac{-3}{1}$ x-int: $(0,0)$ y-int: $(0,0)$

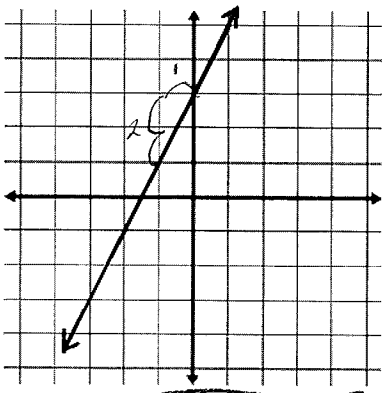
4.) What is the slope of the line That passes through $(0,-1)$ and $(-4,3)$?

$$m = \frac{3 - -1}{-4 - 0} = \frac{4}{-4} = -1$$

Slope-intercept form: $y = mx + b$ (slope, y-int)

5.) Write the equation of each line in slope-intercept form.

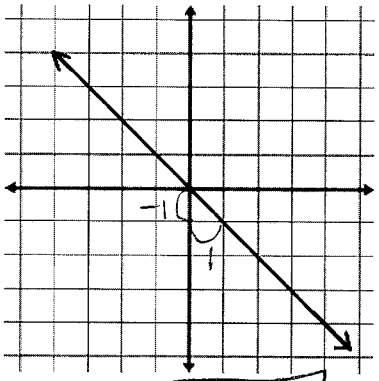
a.)



$m = 2$
$b = 3$

$y = 2x + 3$

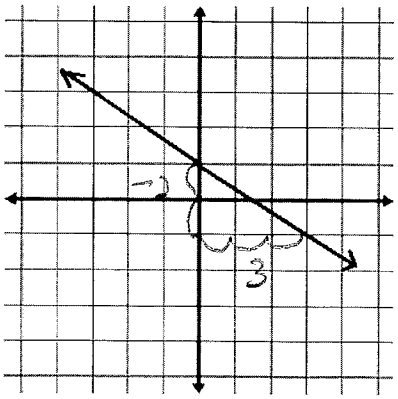
b.)



$b = 0$
$m = 1$

$y = -x$

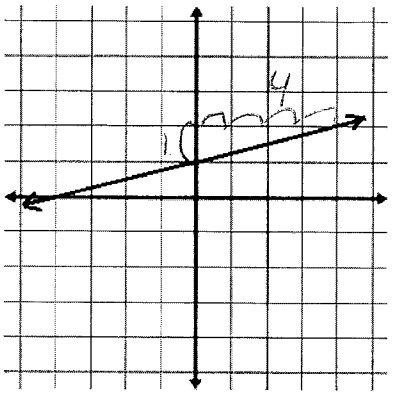
c.)



$b = 1$
$m = -\frac{2}{3}$

$y = -\frac{2}{3}x + 1$

d.)



$b = 1$
$m = \frac{1}{4}$

$y = \frac{1}{4}x + 1$

Rate of Change $= \frac{y_2 - y_1}{x_2 - x_1}$

Rate of Change

Rate of Change – calculates the amount of change in one item divided by the corresponding amount of change in another.
- You are actually finding the *slope* of the 2 quantities.

Remember: Time is usually an x value and money is usually a y value

Examples

1.) Rosemary grows a plan from seed. In 2 weeks (x_1), the plant is 5 cm (y_1) tall. In 6 weeks (x_2), the plant is 17 cm (y_2) tall.

a) What is the average growth rate each week of this plant between weeks 2 and 6?

$\frac{17-5}{6-2} = \frac{12}{4} =$ 3 cm per week

b) If the plant continues to grow at this rate, how tall would it be at week 8?

$y = mx + b$

$5 = 3(2) + b$

$-1 = b$

$y = 3x - 1$

$y = 3(8) - 1$

$= 23$ cm

2.) The chart below shows the cost of renting a car.

Number of Days	Cost to Rent ($)
1	100
2	140
3	180
4	220

+1 → +40, +1 → +40, +1 → +40

$40 more each day

Joey has already rented a car for 4 days, how much more will it cost to rent it for 2 additional days?

a.) $40 (b.) $80 c.) $100 d.) $140

Try One

1.) In 1983 (x_1) a locally-owned car company sold 2,137 (y_1) cars. In 2006 (x_2), the car sales rose to 4,172 (y_2). What was the average rate of change for the total number of cars sold?

$\frac{4172 - 2137}{2006 - 1983} = \frac{2035}{23} =$ 88.48 cars per year

Writing Linear Equations

Slope-Intercept Form: $y = mx + b$ ← m stands for Slope and b is y-intercept

Standard Form: $Ax + By = C$ ← A, B and C are all integers

Writing an Equation from a Table

To write an equation from a table, you are once again going to use **slope-intercept form:** $\boldsymbol{y = mx + b}$

- m is found by calculating: $\frac{change\ in\ y}{change\ in\ x}$ (same as the slope formula).
- b is the y value when x = 0. Sometimes this is given in the table. Other times you will have to find this value on your own.
  To find it on your own:
  - Select a point from the table (corresponding x and y values) and plug the values in for x and y in your equation; you must have m in your equation to do this.
  - Solve for b
  - Plug this value for b into the equation $y = mx + b$

1.) Kara bought a new car and had to take out a car loan. The table below shows the amount Kara still owes (y) after a certain number of months (x).

Money Kara Owes for her Loan:

Months (x)	Money Owed (y)
x_1 4	y_1 \$7,700
8	\$6,800
10	\$6,350
x_2 15	y_2 \$5,225

4 2 5 −900 −450 −1125

Write an equation to show the amount that Kara still owes for her car.

$\frac{5225 - 7700}{15 - 4} = \frac{-2475}{11} = -225$ per month

$y = mx + b$

$5225 = -225(15) + b$

$5225 = -3375 + b$

$+3375 \quad +3375$

$8600 = b$

$y = -225x + 8600$

2.) The Dallastown basketball team is selling t-shirts for their Pink game as a fundraiser. The table below shows the amount of money they raise (y) for a certain amount of t-shirts sold (x).

Money Raised Selling T-Shirts

# T-Shirts Sold	Money Raised ($)
50	$500
60	$620
75	$800
92	$1,004

Write an equation that models the amount of money the team raises for their Pink game.

$\frac{1004-500}{92-50} = \frac{504}{42} = 11.95$ per t-shirt

$y = mx+b$

$1004 = 11.95(92)+b$

$1004 = 1099 + b$

$-95 = b$

$y = 11.95x - 95$

3.) Last summer Tom purchased materials to build model cars and then sold the finished models. He sold each model for the same amount of money. The table below shows the relationship between the number of model cars sold and the running total of Tom's profit.

Tom's Model Car Sales

Model Cars Sold	Total Profit
11	$28
14	$82
16	$118
22	$226

Write a linear equation, in slope-intercept form, to represent the amount of Tom's total profit (y) based on the number of model cars (x) he sold.

$\frac{226-28}{22-11} = \frac{198}{11} = 18$ per car

$226 = 18(22)+b$

$226 = 396 + b$

$-170 = b$

$y = 18x - 170$

Writing an Equation from Points

To write an equation from the slope and a point or two points, you must follow the same steps.

1 – Start with $y = mx + b$
2 – Plug in the slope (m). You will either be given the slope or you will have to find it using the slope formula
3 – Plug in the point for x and y. Once you do this, b is the only variable left in the equation
4 – Solve the equation for b
5 – Write your equation, keeping x and y as variables and substituting your values for m and b.

Examples:

1.) Write the equation of a line that passes through the point $(-3, 8)$ and has a slope of -3.

$x\ y$ m

$8 = -3(-3) + b$
$8 = 9 + b$
$-1 = b$

$y = -3x - 1$

2.) Write the equation of a line that passes through the points $(-2, 2)$ and $(-1, -3)$.

$x_1\ y_1$ $x_2\ y_2$

$m = \frac{-3-2}{-1+2} = \frac{-5}{1}$

$-3 = -5(-1) + b$
$-3 = 5 + b$
$-8 = b$

$y = -5x - 8$

Try Some

3.) Write the equation of a line that passes through the point $(1, 3)$ and has a slope of $-\frac{2}{5}$.

$x\ y$ m

$3 = -\frac{2}{5}(1) + b$
$3\frac{2}{5} = b$

$y = -\frac{2}{5}x + 3\frac{2}{5}$

4.) Write the equation of a line that passes through the points $(-3, 1)$ and $(1, -3)$.

$x_1\ y_1$ $x_2\ y_2$

$m = \frac{-3-1}{1-3} = \frac{-4}{4} = -1$

$-3 = -1(1) + b$
$-2 = b$

$y = -x - 2$

Data Analysis

Data Displays provide a way to study large sets of data more easily and arrive at conclusions more quickly. This process is very helpful in fields like science, medicine, technology, and business.

Types of Data Displays: Stem-and-Leaf Plots, Box-and-Whisker Plots, Histograms, Scatterplots

Scatterplots and Line of Best Fit

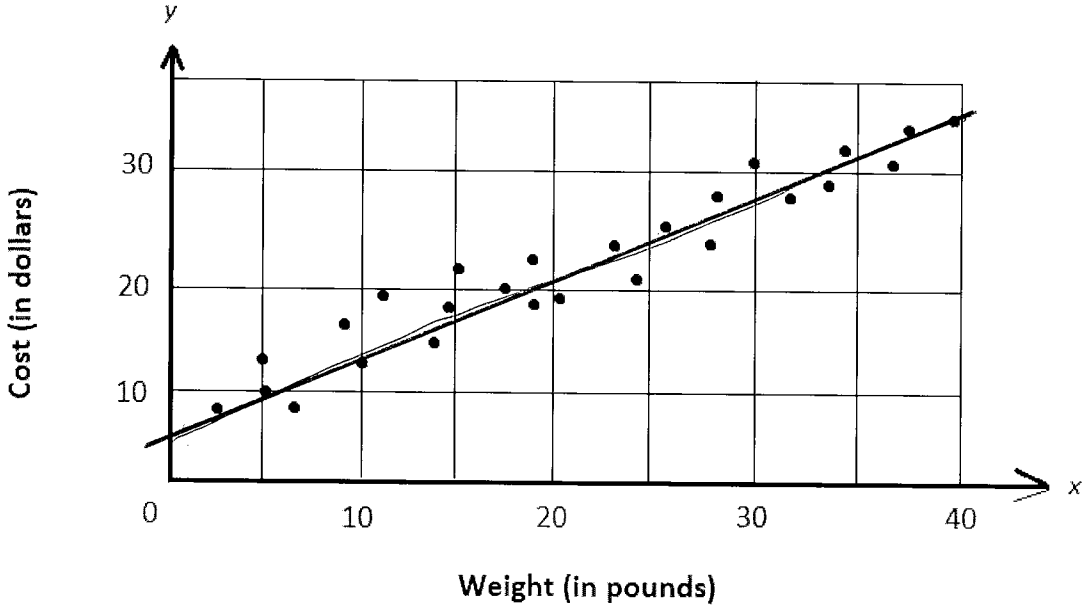
Scatter Plots – provides a picture of the relationship between 2 sets of data. You can use scatter plots to predict relationships and future occurrences.

Correlation – the relationship between the 2 sets of data

Line of Best Fit – the line that represents the general trend of the data. It goes through the middle of the data and shows the trend, it does not have to actually hit the points

** **Note:** If a problem is asking about the line of best fit and there is not one drawn in, you must draw one in yourself.

Ex) The scatter plot below shows the cost (y) of ground shipping packages from Harrisburg, PA, to Minneapolis, MN, based on the package weight (x).



A) Which equation **best** describes the line of best fit?

a.) $y = 0.37x + 1.57$
b.) $y = 0.37x + 10.11$
c.) $y = 0.68x + 2.32$
d.) $y = 0.68x + 6.61$

looking at just the y-int. of each equation only 6.61 makes sense w/ my graph

$y = 0.68(50) + 6.61 = \$40.61$

B) If the trend continues, how much would it cost to send a package that weighs 50 lbs?

Box and Whisker Plots

Quartiles – divide the data into 4 equal parts. Each part contains 25% of the data

Lower Quartile (1st Quartile) – the median of the **lower half** of data

Median (2nd Quartile) – the middle of **all** of the data

Upper Quartile (3rd Quartile) – median of the **upper half** of the data

Interquartile Range – the range from the lower quartile to the upper quartile

Example:

Lower Quartile ↘ Upper Quartile ↓

2, **4**, 7, **8**, 9, **12**, 13

↑ Median

Numbers must be in order from smallest to largest before dividing up the data

Try One

A scientist recorded the temperature, in degrees Celsius, in 12 different parts of rainforest. Her results are shown below:

~~11~~, ~~14~~, ~~12~~, ~~15~~, ~~8~~, ~~16~~, 21, ~~10~~, ~~11~~, ~~17~~, ~~13~~, ~~10~~

8 10 10 11 11 12 13 14 15 16 17 21

10.5 12.5 15.5

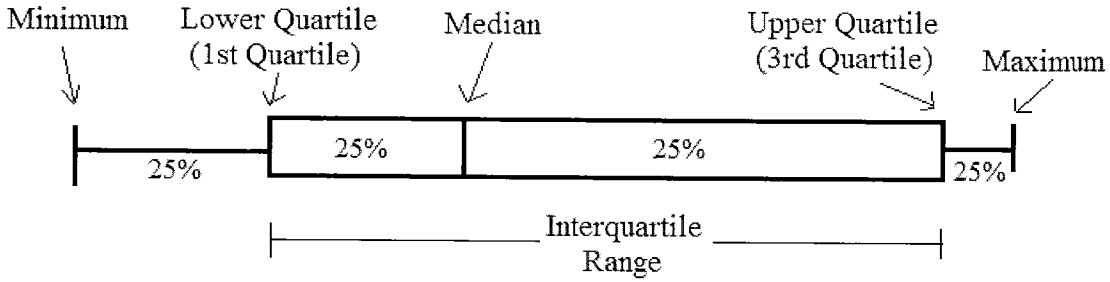
Find the following:

Lower Quartile: 10.5 Median: 12.5 Upper Quartile: 15.5 Interquartile Range: 5

↑

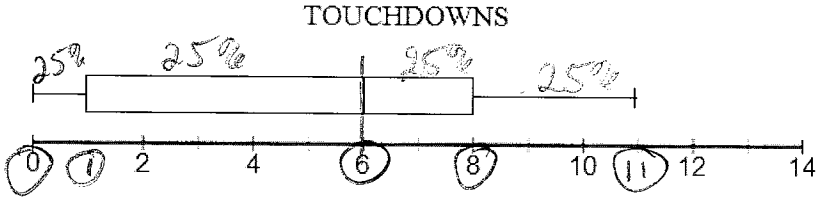
15.5 − 10.5 = 5

Box and Whisker Plot – data display that uses quartiles



Example:

The box-and-whisker plot below shows the number of touchdowns scored each year by a football player.



1.) What is the median number of touchdowns? 6

2.) What is the interquartile range? $8 - 1 = 7$

3.) The upper 25% of the data is between 8 and 11

4.) The middle 50% of the data is between 1 and 8

Stem and Leaf Plots - data display that organizes each piece of data

Ages of Cast Members

1	6 9 9
2	2 4
3	4 5 7
4	
5	7
6	0

median $\frac{34+29}{2} =$

1.) What is the range of the data?

$60 - 16 = 44$

2.) What is the mode of the data?

19

3.) What is the median of the data?

29

4.) What is the 3rd Quartile of the data?

37

16
19
19
22
24 > 29
34
35
37
57
60