Keystone Review
Module 1

The Keystone Exam is broken into 2 Modules.

Module 1	Module 2
• Operations with Real Numbers and Expressions • Linear Equations • Linear Inequalities	• Functions • Coordinate Geometry • Data Analysis

Operations with Real Numbers and Expressions

Under this topic you will be asked to...

- Compare/Order Real Numbers
- Simplify Square Roots
- Find GCF and LCM
- Simplify Expressions Using Exponent Properties
- Simplifying Algebraic Expressions
- Factoring

Linear Equations

Under this topic you will be asked to...

- Write a Linear Equation with One Variable
- Use the Properties of Equality to Solve Linear Equations with One Variable
- Write a Linear Equation with Two Variables
- Write a System of Linear Equations with Two Variables
- Solve a System of Linear Equations with Two Variables

Linear Inequalities

Under this topic you will be asked to...

- Solve a Linear Inequality
- Solve a Compound Inequality
- Solve a System of Linear Inequalities

Writing a Linear Equation with One Variable

Sometimes a linear equation is described in words. You must be able to translate the words into a mathematical equation. In most problems, a consistent amount that does not change will represent the y-intercept. An amount that will change in relation to another quantity represents the slope. We use slope-intercept form $y = mx + b$ to write the equation.

Words that indicate the Slope	Words that indicate the y-intercept
per each every monthly rate	initial amount daily fee starting amount set-up fee

Felix buys a carpet for \$230. The price is \$3.50 per square foot. If Felix had a special discount coupon for \$50 off, which linear equation could be used to find the area, A, of the carpet?

A. $230 = 3.5A + 50$

B. $50 = 3.5A - 230$

C. $230 = 3.5A - 50$

D. $50 = 230 - 3.5A$

Answer:

An online booking agency charges for tickets and includes a ticketing fee for each order. The total charge, c, in dollars, for any number of tickets, t, is described by the function $c = 20t + 4$. Which statement is true?

A. The cost of 20 tickets is \$80.

B. The cost of 4 tickets is \$20.

C. Each ticket costs \$20 and the ticketing fee is \$4.

D. Each ticket costs \$4 and the ticketing fee is \$20.

Answer:

Using Properties of Equality to Solve Linear Equations with One Variable

When solving linear equations, we use the properties of equality.

- Addition Property of Equality
- Subtraction Property of Equality
- Multiplication Property of Equality
- Division Property of Equality

Often we may need to simplify an equation before we solve. You may need to...

- Remove parentheses using the distributive property
- Collect like terms
- Isolate the variable on one side of the equation

The steps Derek used to solve an equation are shown below:

Solve: $0.4x + 5 + 0.2x = 17$

Step 1: $0.4x + 0.2x + 5 = 17$ ← rearranged terms – Commutative
Step 2: $0.6x + 5 = 17$
Step 3: $0.6x = 12$ ← subtraction property – Subtracted 5 from step 2 to get step 3
Step 4: $x = 20$

Which properties justify Step 1 and Step 3?

A. Step 1: Distributive Property
Step 3: Division Property of Equality

B. Step 1: Distributive Property
Step 3: Subtraction Property of Equality

C. Step 1: Commutative Property of Addition
Step 3: Division Property of Equality

(D.) Step 1: Commutative Property of Addition
Step 3: Subtraction Property of Equality

Answer:

What is the solution to the linear equation $\frac{3}{4}x - 5 = 10$?

A. $y = \frac{15}{4}$

B. $y = \frac{20}{3}$

C. $y = \frac{45}{4}$

(D.) $y = 20$

$\frac{3}{4}x - 5 = 10$
$+5 \quad +5$

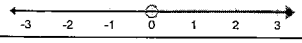
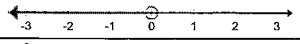
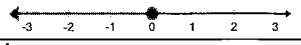
$\frac{4}{3} \cdot \frac{3}{4}x = 15 \cdot \frac{4}{3}$

$x = 20$

Answer:

Solving a Linear Inequality

Solving a linear inequality is the same as solving a linear equation except you **REVERSE** the inequality sign if you are multiplying or dividing by a negative number. The solution set is represented by a graph.

$x > 0$ x is greater than zero	$x < 0$ x is less than zero	$x \geq 0$ x is greater than or equal to zero	$x \leq 0$ x is less than or equal to zero
			

open circle

closed circle

What is the solution to the inequality $-\frac{y}{5} + 6 > 2$?

A. $y < -20$

(B.) $y < 20$

C. $y > -20$

D. $y > 20$

$-\frac{y}{5} + 6 > 2$
$-6 \quad -6$

$5 \cdot \frac{-y}{5} > -4 \cdot 5$

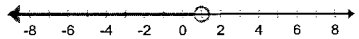
divide by neg. so inequality switches →

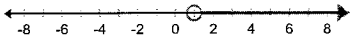
$\frac{-y}{-1} > \frac{-20}{-1}$

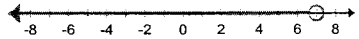
$y < 20$

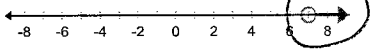
Answer:

Which graph shows the solution set to the inequality $3(3-x) < -12$?

A. 

B. 

C. 

D. 

Answer:

$3(3-x) < -12$

$+9 \quad +9$

$-3x < -12$

$\frac{-3x}{-3} < \frac{-21}{-3}$ ← divide by neg.

So switch ineq.

$x > 7$

↖ not = so open circle

Recall the phrases that represent the different inequality symbols.

$>$	$\geq$	$<$	$\leq$
is more than is greater than is larger than above	minimum at least is not less than no less than not smaller than	is smaller than is less than is fewer than below	maximum at most is not more than no more than is not greater than

Write the inequality that models the situation. Solve and then interpret your solution.

Ore Valley Elementary school is having a fall carnival. Admission into the carnival is \$3 and each game inside the carnival costs \$0.25. Tanner has no more than \$10 to spend. What is the maximum number of games he can play?

Answer:

$3 + 0.25g \leq 10$

$-3 \quad -3$

$\frac{0.25g}{0.25} \leq \frac{7}{0.25}$

$g \leq 28$ games

He can play up to 28 games and still keep to his \$10 limit.

Rebecca wants to order some DVDs from Amazon. The DVDs cost \$7.98 each and there is a shipping charge of \$5. She can spend no more than \$100. What is the maximum number of DVDs she can buy?

Answer:

inequality → $7.98d + 5 \leq 100$

$-5 \quad -5$

$\frac{7.98d}{7.98} \leq \frac{95}{7.98}$

Solution → $d \leq 11.9$

She can buy at most 11 DVDs to stay inside her budget

↑

interpret

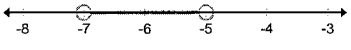
Solving a Compound Inequality

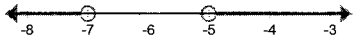
A compound inequality is formed when two inequalities are joined by the words **"and"** or **"or"**.

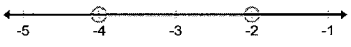
Conjunction	Disjunction
AND	OR
The solution must satisfy **both** inequalities for the conjunction to be true.	The solution must satisfy either of the inequalities for the disjunction to be true.
Look for the **overlap** or intersection in the graph of the two inequalities to be our solution.	The two inequalities **together** on the same graph or the union of the graphs will be our solution.

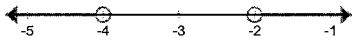
↖ Conjunction when x is "Sandwiched" between two inequalities

Which graph shows the solution to $7 < -3 - 2x < 11$?

(A.) 

B. 

C. 

D. 

Answer:

$7 < -3 - 2x < 11$

$-3 \quad -3 \quad -3$

$4 < -2x < 8$

divide by a neg. switch signs → $\frac{4}{-2} < \frac{-2x}{-2} < \frac{8}{-2}$

both

Solve both inequalities together doing the same to both sides

$-2 < x < 4$

What is the solution to the compound inequality $5 + 3x < 2$ or $2x - 7 > -3$?

(A.) $x < -1$ or $x > 2$

B. $-1 < x < 2$

C. $x < -1$ or $x > -5$

D. $-1 < x < -5$

Answer:

Solve each inequality seperately then find solution

$5 + 3x < 2$ and $2x - 7 > -3$

$-5 \quad -5$ $\qquad$ $+7 \quad +7$

$\frac{3x}{3} < \frac{-3}{3}$ $\qquad$ $\frac{2x}{2} > \frac{4}{2}$

$x < -1$ $\qquad$ $x > 2$

Writing a Linear Equation with Two Variables

Jackson has 75¢ in dimes, d, and nickels, n, in his pocket. Which equation could be solved to find the possible combinations of dimes and nickels Jackson has?

A. $75 = d + n$

B. $75 = dn$

C. $75 = 10d \cdot 5n$

(D.) $75 = 10d + 5n$

Answer:

dimes = 10 cents each

nickels = 5 cents each

$10d + 5n = 75$

The growth of a kitten is described by the equation $y = 2.5x + 4$, where y represents the kitten's weight in ounces x weeks after it was born. What is the meaning of the fact that the point $(4, 14)$ lies on the graph of the equation?

x = weeks old y = kitten's weight

A. The kitten had an initial weight of 4 ounces.

B. The kitten is growing at a rate of 4 ounces per week.

C. The kitten weighed 4 ounces when it was 14 weeks old.

(D.) The kitten weighed 14 ounces when it was 4 weeks old.

Answer:

Writing a System of Linear Equations with Two Variables

Georgia needs to print posters for a community event. Stellar Printers will charge Georgia a $300 set-up fee plus $1 per poster. Artemis Printers will charge Georgia a $200 set-up fee plus $1.50 per poster. Write a system of linear equations where y is the cost in dollars and x is the number of posters.

A. $\begin{cases} y = 300x \\ y = 200x + 1.5 \end{cases}$

(B.) $\begin{cases} y = x + 300 \\ y = 1.5x + 200 \end{cases}$

C. $\begin{cases} y = x - 300 \\ y = 1.5x - 200 \end{cases}$

D. $\begin{cases} x = y + 300 \\ x = 1.5y + 200 \end{cases}$

Answer:

Stellar $300 + 1x = y$

Artemis $200 + 1.50x = y$

per poster cost needs to be attached to # of posters (x) and the fee gets added on

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Stephen bought a total of 8 pounds of peanuts and cashews. Peanuts, p, cost $2 per pound and cashews, c, cost $5 per pound. The total amount Stephen spent on the peanuts and cashews was $25. Which system of equations could be solved to find how many pounds of peanuts Stephen bought?

(A.) $\begin{cases} 2p + 5c = 25 \\ p + c = 8 \end{cases}$

B. $\begin{cases} 5p + 2c = 25 \\ p + c = 8 \end{cases}$

C. $\begin{cases} 2p + 5c = 8 \\ p + c = 25 \end{cases}$

D. $\begin{cases} 2p = 8 \\ 5c = 25 \end{cases}$

p = pounds of peanuts

c = pounds of cashews

Answer:

all dollar amounts should be in one equation $2p + 5c = 25$

second equation is based on total pounds bought $p + c = 8$

Solving a System of Linear Equations with Two Variables

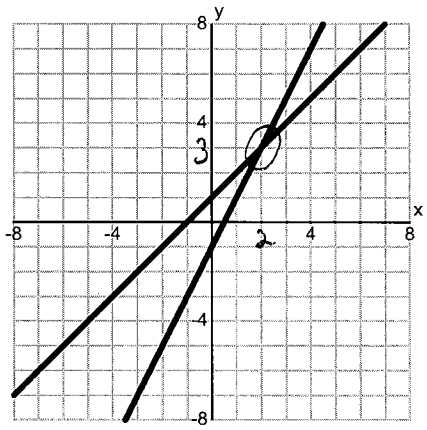
A System of Linear Equations is two or more linear equations with multiple variables.
The solution of a system of linear equations is the point where the two lines intersect.
We learned three ways to solve a system of linear equations.

1. **Graphing**
2. **Substitution**
3. **Elimination**

Solving by Graphing

When in the system of linear equations is graphed, the solution is the point of intersection of the two lines.

What is the solution of the system of equations graphed below?



(2, 3)

A. $(0, -1)$

B. $(0, 1)$

(C.) $(2, 3)$

D. $(3, 2)$

Answer: just identify the x + y values where they intersect

Solving by Substitution

1. Solve one of the equations for one of its variables.
2. Substitute the expression from step 1 into the OTHER equation and solve for the other variable.
3. Substitute the value from step 2 into the one of the original equations and find the value of the second variable.

What is the solution to the system of equations shown below?

$$\begin{cases} 2x + y = 9 \\ x = 3 - y \end{cases}$$

A. $(1, 7)$

B. $(4, 1)$

C. $(2, 5)$

(D.) $(6, -3)$

Answer:

plug the answer for y into the x = equation → $X = 3 + +3 = 6$

$(6, -3)$

Since $x = 3 - y$ plug $3 - y$ in for x in the 1st equation

$2(3 - y) + y = 9$

$6 - 2y + y = 9$

$6 - y = 9$

$-6 \quad -6$

$+y = 3$

$-1 \quad -1$

$y = -3$

Solving by Elimination

1. Multiply one or both of the equations by a number so that the coefficients of one of the variables are opposites
2. Add the equations from step 1 together. Combining the equations will eliminate one of the variables.
3. Solve for the remaining variable.
4. Substitute the value from step 3 into the one of the original equations and find the value of the second variable.

What is the solution to the system of equations shown below?

$$\begin{cases} 4x - y = 7 \\ -4x - 2y = 2 \end{cases}$$

A. $(-1, 3)$ B. $(1, -3)$

C. $(2, -3)$ D. $(3, 5)$

Answer:

Can just add equations since the x's are already exact opposites & cancel each other out

$4x - y = 7$
$-4x - 2y = 2$
$-3y = 9$
$\frac{-3y}{-3} = \frac{9}{-3}$
$y = -3$

plug answer for y into either equation

$4x + 3 = 7$
$-3 \quad -3$
$\frac{4x}{4} = \frac{4}{4}$
$x = 1$

Solving a System of Linear Inequalities

A system of linear inequalities is graphed on the same coordinate plane. The solutions of this problem are the points located in the **overlap** of the shaded region.

If the inequality sign is $<$ or $>$ then the line is dashed. ←- - - - - - - - - - - - - - - -→	If the inequality sign is $\leq$ or $\geq$ then the line is solid. ←——————→

Graph the solution of the system of linear inequalities $\begin{cases} x + 2y > -6 \\ x \geq -2 \end{cases}$ ← vertical line

Solid

Shade in direction of arrows for x lines

get y by itself 1st

$x + 2y > -6$
$-x \quad -x$

$\frac{2y}{2} > \frac{-x}{2} - \frac{6}{2}$

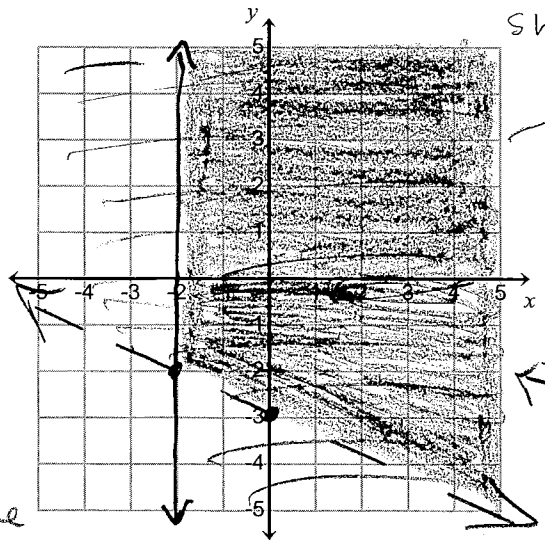
$y > -\frac{1}{2}x - 3$

slope y-intercept

up 1
left 2

dotted line

$y >$ Shade above line



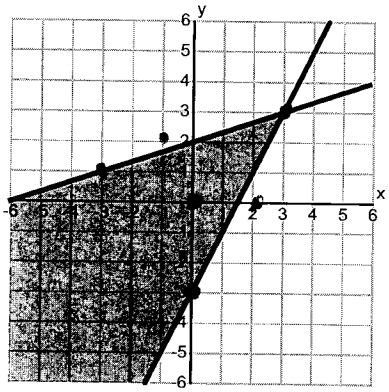
Check:
(0,0) is in shaded region
$0 + 2(0) > -6$ $0 > -6$ ✓
$0 \geq -2$ ✓
both check

area where it is above dotted line and right of solid line

to be solutions they must be in shaded region or on a solid line.

State whether or not the given points are solutions of the system of linear inequalities below.

1. (0, 0) yes
2. (−1, 2) no
3. (0, −3) yes

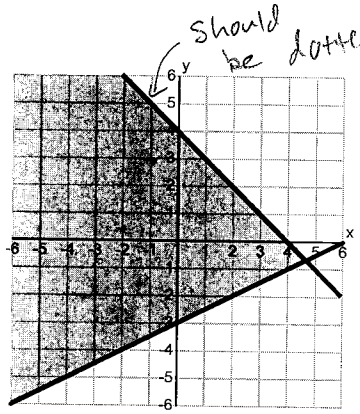


4. (−3, 1) yes
5. (2, 0) no
6. (3, 3) yes

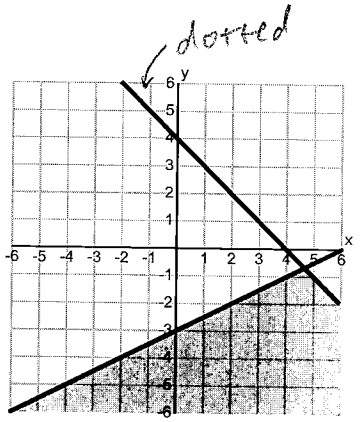
Which graph represents the solution of the system of inequalities shown below?

$$\begin{cases} y \geq 2x - 3 \\ x + y < 4 \end{cases}$$

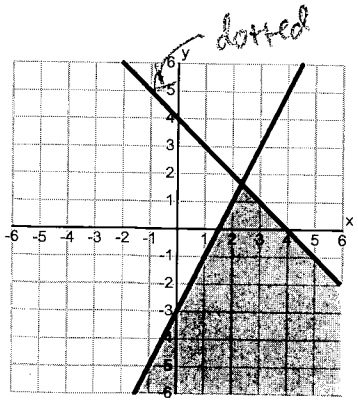
A.



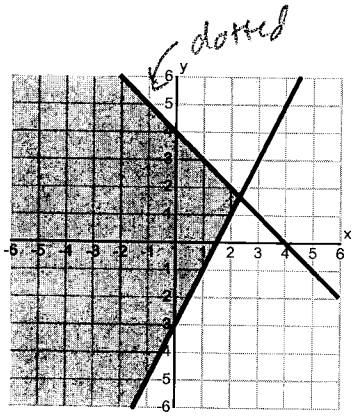
B.



C.



D.



test a point (0,0)

$0 \geq 2(0) - 3$

$0 \geq 0 - 3$ ✓

$0 + 0 < 4$

$0 < 4$ ✓

So (0,0) is a solution which eliminates B+C

Check (3,0)

$0 \geq 2(3) - 3$

$0 \geq 6 - 3$

$0 \geq 3$ no

So (3,0) is not a solution which eliminates A

Application of a System of Linear Equations

In Lewis Carroll's "Through the Looking Glass" Tweedledum says, "The sum of your weight and twice mine is 361 pounds." Tweedledee replies, "The sum of your weight and twice mine is 362 pounds." Write a system of linear equations and find both of their weights.

Let x = Tweedledee's weight
Let y = Tweedledum's weight

Answer: $-2(x + 2y = 361)$

Tweedledee (x) weighs 121 pounds
Tweedledum (y) weighs 120 pounds

Solving by elimination

$$\begin{array}{r} -2x + y = 362 \\ -2x - 4y = -722 \\ \hline -3y = -360 \end{array}$$

$$\frac{-3y}{-3} = \frac{-360}{-3}$$

$y = 120 \quad x = 121$

$x + 2(120) = 361$

$x + 240 = 361$

$-240 \quad -240$

Lucy tells her little brother Jack that she is holding 20 coins consisting of dimes and quarters. They have a total value of $4.10. She tells Jack that she will give him the coins if he can tell her how many of each she is holding. Solve this problem for Jack.

Let d = number of dimes
Let q = number of quarters

Answer: value of dimes 0.10 + value of quarters 0.25 need to be in an equation with total value
second equation is for # of coins

$0.10d + 0.25q = 4.10$

Solving by Substitution

$d + q = 20$

$d + q = 20$

$-q \quad -q$

$d = 20 - q$

$0.10(20 - q) + 0.25q = 4.10$

$2 - 0.10q + 0.25q = 4.10$

$2 + 0.15q = 4.10$

$-2 \quad -2$

$\frac{0.15q}{.15} = \frac{2.10}{.15}$

$q = 14 \quad d + 14 = 20$

$d = 6$

She has 14 quarters and 6 dimes

maximum amount

Application of a System of Linear Inequalities

A clothing manufacturer has 900 yards of cotton to make shirts and pajamas. A shirt requires 1 yard of fabric and a pair of pajamas requires 2 yards of fabric. It takes 2 hours to make a shirt and 3 hours to make the pajamas, and there are 1200 hours available to make the clothing.

a. Write a system of linear inequalities where x is the number of shirts and y is the number of pajamas.

One inequality should relate all yards measures $1x + 2y \leq 900$

other inequality should relate all time measures $2x + 3y \leq 1200$

b. Graph the solution. Be sure to label your axes. get y by itself

$2x + 3y \leq 1200$

$-2x \quad -2x$

$\frac{3y}{3} \leq \frac{-2x}{3} + \frac{1200}{3}$

$y \leq -\frac{2}{3}x + 400$

$-$ down 2 / $+$ right 3 ; y-int

Solid line

$\leq$ Shade below line

$1x + 2y \leq 900$

$-x \quad -x$

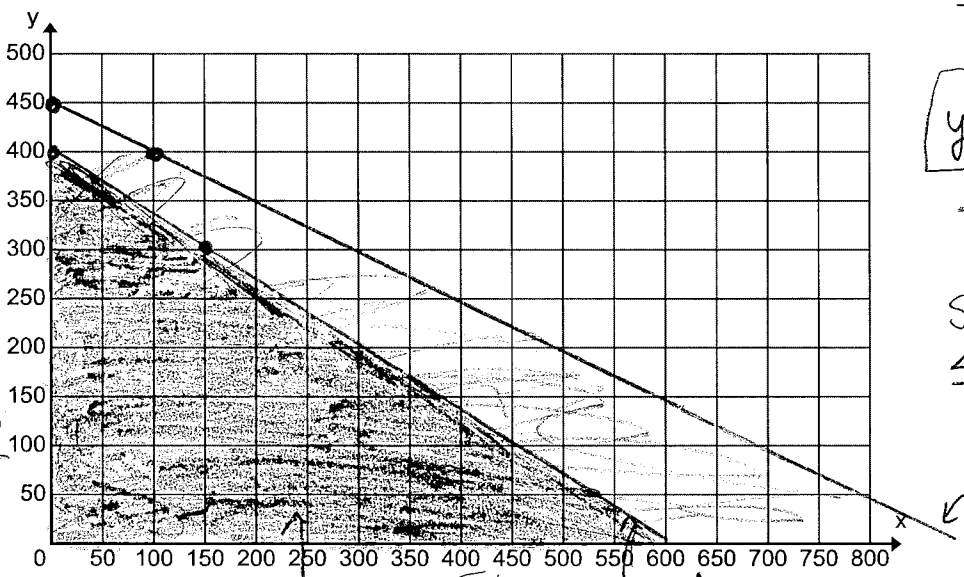
$\frac{2y}{2} \leq \frac{-x}{2} + \frac{900}{2}$

$y \leq -\frac{1}{2}x + 450$

$-$ down 1 / $+$ right 2 ; y-int.

Solid line

$\leq$ Shade below line



1st inequality

Shirt

2nd inequality

region that is < both

c. How many shirts and pajamas can be manufactured? Give three solutions.

Can be any 3 points in shaded region

ex: 250 shirts + 150 pajamas (250, 150)

50 shirts + 300 pajamas (50, 300)

400 shirts + 100 pajamas (400, 100)

maximum seating ↖ x y

Zach's high school theater can seat at most 400 people. Adult tickets are \$5 and student tickets are \$2. How many adult tickets and student tickets must be sold so that the school makes at least \$1,000? Let x = number of adult tickets sold and y = number of student tickets sold.

minimum amount ↗

a. Write a system of inequalities to represent the problem.

1st inequality should relate all dollar amounts $5x + 2y \geq 1000$

2nd inequality should describe the # of people $x + y \leq 400$

b. Graph the solution. Be sure to label your axes.

get y by itself

$x + y \leq 400$

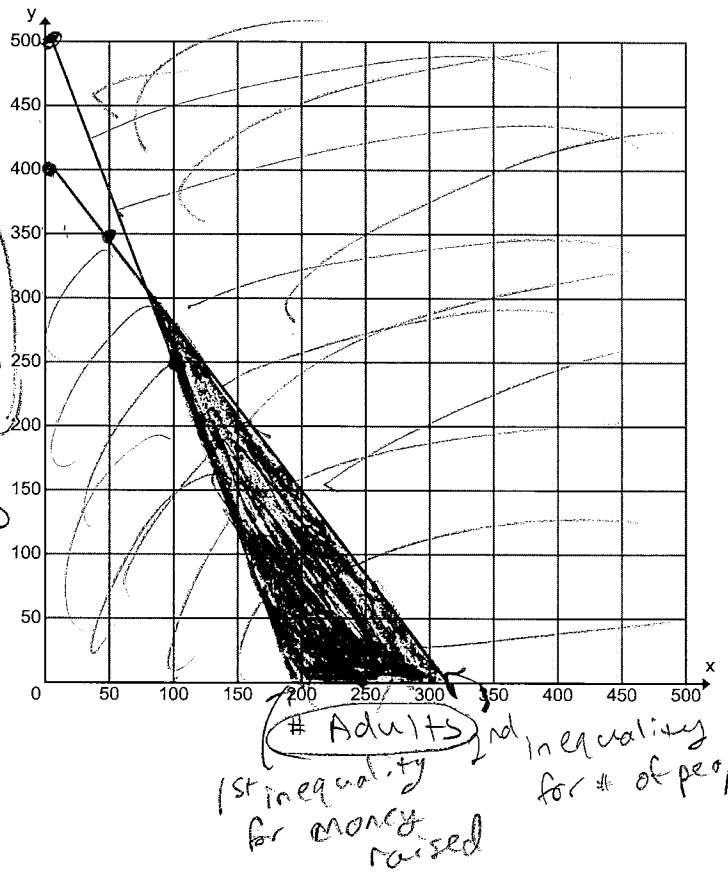
$-x \quad -x$

$y \leq -x + 400$

slope: - down 1 / + right 1 ; y-int.

Solid line

$\leq$ Shaded below



1st inequality for money raised

2nd inequality for # of people

$5x + 2y \geq 1000$

$-5x \quad -5x$

$\frac{2y}{2} \geq \frac{-5x + 1000}{2}$

$y \geq -\frac{5}{2}x + 500$

slope: - down 5 / + right 2 ; y-int.

Solid line

$\geq$ Shade above

c. Give three possible combinations of adult and student tickets that could be sold so the school meets its goal.

pick any 3 values in shaded region or on the lines where we are less than 1st inequality and greater than 2nd inequality

ex:

200 adults + 100 students (200, 100)

150 adults + 150 students (150, 150)

100 adults + 250 students (100, 250)