Unit 11 Notes

What is Greatest Common Factor (GCF)?

Integers	The greatest number that is a factor of all the integers
Monomials	The product of their common factors when each monomial is expressed in factored form

If two or more integers or monomials have no common factors (GCF = 1) then they are called **relatively prime**.

Examples

Find the GCF of each.

1. 18 and 24

3. $6p^2q^3$ and $9p^5q$

2. $3x^2$ and $9x^3$

4. $12x^3$; $8x^7$ and $24x^2$

	Integers	Variables
What do I look for to find the GCF of Monomials?		

Try These

Find the GCF of each.

1. $4c^3$ and $8c$

   1. _____

2. $10b$ and $25b^5$

   2. _____

3. $45a^3b$ and $35a$

   3. _____

4. $30a^2$ and $13b$

   4. _____

5. $7xy^3$ and $15x^2y^2$

   5. _____

6. $42ab^2c^3$ and $30a^3b^2c$

   6. _____

7. $8a^3b^2$; $14a^2b$; $21a^2b^5$

   7. _____

8. $20ax^2$; $32a^3x$ and $40a^6$

   8. _____

9. $25p^3qr^6$; $15p^4q^3$ and $65pq^2r^4$

   9. _____

10. $28x^3y^5z^2$; $21x^5y^2z$ and $56x^3yz^6$

   10. _____

Factoring by GCF

What is Factoring?
- ➤ Factoring is writing a polynomial expression in terms of its factors
- ➤ We are 'Un-doing' the distributive property
- ➤ **WHY do we need to factor?** It helps to simplify algebraic fractions and solve polynomial equations

Factoring using the Greatest Common Factor (GCF)
- ➤ First, find the GCF of all of the terms in the polynomials (like Objective 1)
- ➤ Next, factor that GCF out of each term using division
- ➤ **Check:** If we use distributive property to multiply, we should go back to the original expression

Examples

Factor each polynomial by finding the GCF.

1. $6x^2 + 15x$

2. $16y^2 - 24y + 32$

3. $-12a^4 + 24a^2 - 6a$

4. $5x^3y^2 - 15x^2y - 30xy$

Try These

Factor each polynomial by finding the GCF.

1. $3x^2 - 3x$ 1. _____

2. $-3x^3 - 33x$ 2. _____

3. $15x^2 + 18x$ 3. _____

4. $19w^5 - 19w^2$ 4. _____

5. $36x^3 - 24x^2 + 8x$ 5. _____

6. $2a^4 - 10a^2 + 2a$ 6. _____

7. $3x^3 + 10x^2 - x$ 7. _____

8. $-10x^6 + 12x^5 - 4x^4$ 8. _____

9. $2x^2y^2 - 4x^3y$ 9. _____

10. $27ab^3 - 36ab^2 + 18ab$ 10._____

11. $8x^4y^2 + 12x^3y^3 - 16x^2y^4$ 11._____

12. $36a^5b^3 - 48a^4b^4 + 72a^6b^2$ 12._____

Factoring a Difference of Squares

Using FOIL, multiply the following.

1. $(x-3)(x+3)$

   1. _____

2. $(y+5)(y-5)$

   2. _____

3. $(2w+3)(2w-3)$

   3. _____

4. $(7n-2)(7n+2)$

   4. _____

5. $(4-g)(4+g)$

   5. _____

6. $(a^2-b)(a^2+b)$

   6. _____

7. $(3x+5y)(3x-5y)$

   7. _____

8. $(10m+n^3)(10m-n^3)$

   8. _____

What do you notice about the two binomials that you are multiplying?

What do you notice about the product of the two binomials?

Try These

Factor each polynomial as a difference of squares.

1. $x^2 - 16$ 1. _____

2. $p^2 - 144$ 2. _____

3. $w^2 - 1$ 3. _____

4. $49 - y^2$ 4. _____

5. $25 - a^2$ 5. _____

6. $4k^2 - 121$ 6. _____

7. $x^2 y^2 - 1$ 7. _____

8. $81x^2 - 16y^2$ 8. _____

9. $16r^2 - s^2$ 9. _____

10. $9n^2 - 64m^2$ 10. _____

First, factor out a GCF and then factor as a difference of squares.

11. $2x^3 - 128x$ 11. _____

12. $27x^2 - 9$ 12. _____

13. $49m^2 n^2 - n^4$ 13. _____

Factoring $x^2 + bx + c$

Multiply each polynomial, using distributive property, and write the answer in standard form.

1. $(x + 5)(x + 4)$

2. $(x + 6)(x + 1)$

3. $(x - 2)(x - 4)$

Can you use the pattern you noticed above to factor $x^2 + 9x + 18$?

Factoring a trinomial in the form $x^2 + bx + c$ or $x^2 - bx + c$

Find 2 numbers that _____

and _____

Their signs will be _____

Examples

Factor each polynomial completely.

1. $x^2 + 10x + 21$ What are we looking for?   2 Numbers that: add to _____
    and multiply to _____

2. $x^2 - 7x + 12$ What are we looking for?   2 Numbers that: add to _____
    and multiply to _____

3. $x^2 + 8xy + 16y^2$ What are we looking for?   2 Numbers that: add to _____
    and multiply to _____

Try These

Factor each polynomial completely.

1. $x^2 + 3x + 2$ 1. _____

2. $x^2 - 6x + 5$ 2. _____

3. $x^2 - 18x + 81$ 3. _____

4. $x^2 + 11x + 30$ 4. _____

5. $x^2 - 9x + 18$ 5. _____

6. $x^2 + 12xy + 36y^2$ 6. _____

7. $x^2 - 16x + 48$ 7. _____

8. $x^2 + 27xy + 50y^2$ 8. _____

9. $x^2 - 30x + 144$ 9. _____

10. $x^2 + 25xy + 24y^2$ 10. _____

Multiply each polynomial, using distributive property, and write your answer in standard form.

1. $(x - 5)(x + 4)$

2. $(x + 6)(x - 1)$

3. $(x - 2)(x + 4)$

Can you use the pattern you notice above to factor $x^2 + 2x - 15$?

Factoring a trinomial in the form $x^2 + bx - c$

Find 2 numbers that _____

and _____

Their signs will be _____

Examples

Factor each polynomial completely.

1. $x^2 - x - 12$ What are we looking for?   2 Numbers that: add to _____
                                                and multiply to _____

2. $x^2 + 7x - 8$ What are we looking for?   2 Numbers that: add to _____
                                                and multiply to _____

3. $x^2 - 6xy - 16y^2$ What are we looking for?   2 Numbers that: add to _____
                                                    and multiply to _____

4. $x^2 + xy - 42y^2$ What are we looking for?   2 Numbers that: add to _____
                                                and multiply to _____

Try These

Factor each polynomial completely.

1. $x^2 - x - 2$

1. _____

2. $x^2 + 2x - 3$

2. _____

3. $x^2 - 2x - 35$

3. _____

4. $x^2 + 8x - 20$

4. _____

5. $x^2 - 5x - 24$

5. _____

6. $x^2 + 9x - 36$

6. _____

7. $x^2 - 3xy - 4y^2$

7. _____

8. $x^2 + 4x - 32$

8. _____

9. $x^2 - 11xy - 12y^2$

9. _____

10. $x^2 + 3xy - 72y^2$

10. _____

Putting it all together…

Factoring $x^2 + bx + c$	
Sign Pattern	Numbers
If c is positive ( )( ) ( )( )	2 numbers that **add to** _____
If c is negative ( )( )	**multiply to** _____

Try These

Factor each polynomial completely.

1. $x^2 + 16x + 28$

   1. _____

2. $y^2 - 4y - 21$

   2. _____

3. $a^2 + 15a - 16$

   3. _____

4. $x^2 - 6x + 9$

   4. _____

5. $m^2 + 19mn + 18n^2$

   5. _____

6. $z^2 - 5z - 12$

   6. _____

7. $a^2 - 3ab - 10b^2$

   7. _____

8. $y^2 + 10y + 25$

   8. _____

9. $x^2 - 16$ (Think $x^2 + 0x - 16$)

   9. _____

10. $n^2 - n - 20$

   10. _____

Factoring using Different Methods

This objective is a mix of the different problems you have seen in objectives 2 through 4. If the polynomial cannot be factored, write PRIME.

1. $6x^2 - 24x$

1. _____

2. $x^2 - x - 20$

2. _____

3. $x^2 - 8x + 16$

3. _____

4. $x^2 - 9$

4. _____

5. $x^2 - 9x + 8$

5. _____

6. $x^2 - 3x - 8$

6. _____

7. $4x^3y + 16x^2y + 4xy^2$

7. _____

8. $x^2 - 10x - 24$

8. _____

9. $49x^2 - 1$

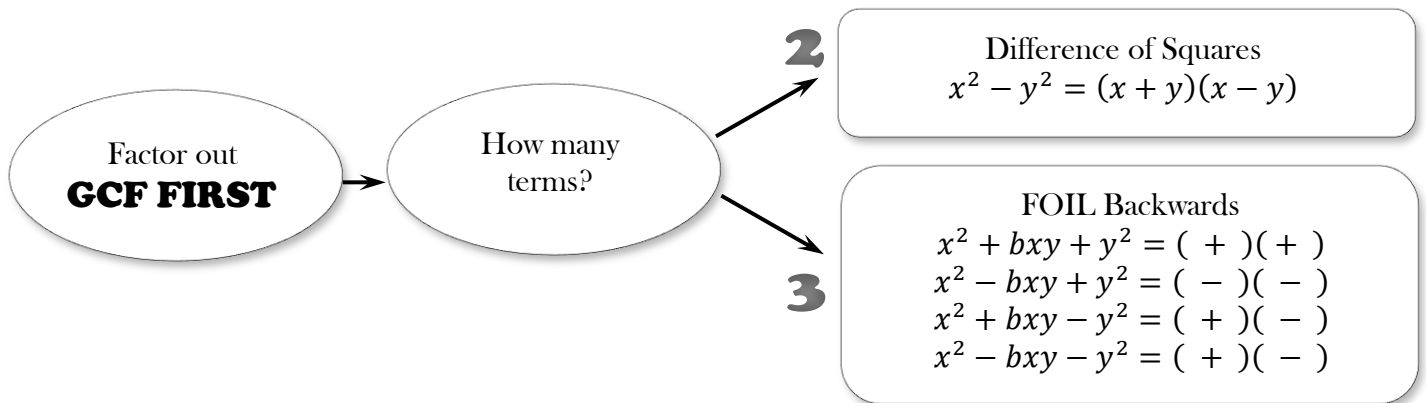9. _____

10. $x^2 + 10xy + 24y^2$

10. _____

11. $x^2 - 100y^2$

11. _____

12. $x^3 + 5x^2 + x$

12. _____

Two-Step Factoring

Factor out
GCF FIRST

How many terms?

2 Difference of Squares
$$x^2 - y^2 = (x + y)(x - y)$$

3 FOIL Backwards
$$x^2 + bxy + y^2 = (\ +\ )(\ +\ )$$
$$x^2 - bxy + y^2 = (\ -\ )(\ -\ )$$
$$x^2 + bxy - y^2 = (\ +\ )(\ -\ )$$
$$x^2 - bxy - y^2 = (\ +\ )(\ -\ )$$

Examples

Factor completely.

1. $4x^3 - 36x$

2. $4x^2 - 12x + 8$

3. $5x^4 - 10x^3 - 75x^2$

4. $3x^3y^2 - 147xy^2$

Try These

5. $2x^2 + 6x + 4$

5. _____

6. $2x^3 + 8x^2 - 64x$

6. _____

7. $a^2x^2 - 16a^2$

7. _____

8. $7x^2 - 14x - 21$

8. _____

9. $x^4 + 3x^2 - 10x^2$

9. _____

10. $4x^2 - 16$

10. _____

11. $9x^2 - 54x + 72$

11. _____

12. $3x^3 - 9x^2 - 54x$

12. _____

13. $5x^2y^3 - 180x$

13. _____

14. $x^4y - 15x^3y + 56x^2y$

14. _____

15. $72y - 2x^2$

15. _____