Unit 9 Notes

Perfect Squares

List the perfect squares up to 15^2.

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2	13^2	14^2	15^2
1														*225*

Estimating Square Roots

Approximate $\sqrt{78}$ to the nearest tenth.

Find the two perfect squares that 78 lies between.

$$\_\_\_\_\_\_ < \sqrt{78} < \_\_\_\_\_\_$$

$$\sqrt{64} < \sqrt{78} < \sqrt{81}$$

$$8 < \ ? \ < 9$$

The $\sqrt{78}$ is between 8 and 9. To find the tenth, $\frac{\text{distance from 64 to 78}}{\text{distance from 64 to 81}} = \frac{14}{17} \approx 0.8$

So, $\sqrt{78}$ can be estimated as approximately _________ .

Examples

1. Approximate $\sqrt{45}$ to the nearest tenth.

2. Approximate $\sqrt{130}$ to the nearest tenth.

Simplifying Square Roots

Simplifying a square root of a whole number means finding an equivalent expression with the smallest possible number under the radical sign. This is called writing the number in **simplest radical form**.

To write a square root in simplest radical form we first find the **largest possible square factor** of the number under the radical sign. Then we use the Product property.

Product Property of Square Roots

The Product rule states that we can break the square root a number into the square root of product of two factors.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

Examples

Simplify $\sqrt{20}$.

Look for the largest possible perfect square that will divide evenly into 20. 4, 9 and 16 are perfect squares less than 20 but, 4 is the only one that will divide evenly into 20. To simplify, we write 20 as a multiplication problem with 4 being one of the factors.

$$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$$

The simplest radical form of $\sqrt{20}$ is __________ .

Examples

Rewrite each square root in simplest radical form.

1. $\sqrt{18} =$
2. $\sqrt{80} =$
3. $\sqrt{98} =$
4. $\sqrt{200} =$
5. $\sqrt{63} =$
6. $\sqrt{288} =$

Try These

Rewrite each square root in simplest radical form.

1. $\sqrt{8} =$
2. $\sqrt{32} =$
3. $\sqrt{24} =$
4. $\sqrt{50} =$
5. $\sqrt{500} =$
6. $\sqrt{162} =$
7. $\sqrt{48} =$
8. $\sqrt{108} =$

Adding and Subtracting Square Roots

We can add and subtract $5x$ and $2x$ because they are like terms. To add and subtract radicals, the radicands (the number under the radical sign) must be the same. We call these **like radicals.**

$$5\sqrt{6} + 2\sqrt{6} = 7\sqrt{6}$$
$$5\sqrt{6} - 2\sqrt{6} = 3\sqrt{6}$$

Examples

1. $\sqrt{12} + \sqrt{75}$

   At first it doesn't look like these two radicals will add. But we must simplify them first to see if they are indeed like radicals.

$$\sqrt{12} + \sqrt{75}$$
$$\sqrt{4 \cdot 3} + \sqrt{25 \cdot 3}$$
$$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$$

2. $\sqrt{8} + \sqrt{18}$

3. $\sqrt{500} - \sqrt{20}$

4. $\sqrt{12}+\sqrt{27}$

5. $\sqrt{6}-\sqrt{24}-\sqrt{54}$

Multiplying Square Roots

When simplifying radicals we use the Product property that says $\sqrt{ab}=\sqrt{a}\cdot\sqrt{b}$.
By writing the Product property the other way around we see that we can use it to multiply square roots as well as to find them. $\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$

Examples

1. $\sqrt{3}\cdot\sqrt{6}$

   Multiply the square roots. $\sqrt{3}\cdot\sqrt{6}=\sqrt{18}$
   Now, see if the answer will simplify. Can we simplify the $\sqrt{18}$?
   $\sqrt{18}=\sqrt{9}\cdot\sqrt{2}=3\sqrt{2}$

   So, $\sqrt{3}\cdot\sqrt{6}=3\sqrt{2}$

2. $\sqrt{10}\cdot\sqrt{20}$

3. $3\sqrt{2}\cdot\sqrt{72}$

4. $3\sqrt{6}\cdot 4\sqrt{8}$

5. $\left(3\sqrt{5}\right)^2$

6. $\sqrt{2}\left(\sqrt{6}+3\right)$

Try These

Simplify the following radicals.

1. $\sqrt{3} \cdot \sqrt{12}$ 1. ______________________

2. $\sqrt{5} \cdot \sqrt{10}$ 2. ______________________

3. $2\sqrt{8} \cdot \sqrt{6}$ 3. ______________________

4. $\sqrt{2} \cdot 3\sqrt{6} \cdot 2\sqrt{8}$ 4. ______________________

5. $2\sqrt{6} \cdot 5\sqrt{24}$ 5. ______________________

6. $(2\sqrt{3})^2$ 6. ______________________

7. $\sqrt{15} \cdot 6\sqrt{3}$ 7. ______________________

8. $\sqrt{3}(5 - \sqrt{3})$ 8. ______________________

9. $\sqrt{20} \cdot \sqrt{16} \cdot \sqrt{8}$ 9. ______________________

10. $(-3\sqrt{6})^2$ 10. ______________________

Dividing Square Roots

An expression that contains a radical is in simplest radical form if…

❶

❷

❸

Quotient Rule for Radicals

The quotient rule for radicals states…

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad \text{and} \quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Simplify $\sqrt{\frac{36}{25}}$

Using the quotient rule we can rewrite $\sqrt{\frac{36}{25}}$ as $\frac{\sqrt{36}}{\sqrt{25}}$ and then simplify.

$$\sqrt{\frac{36}{25}} = \frac{\sqrt{36}}{\sqrt{25}} = \frac{6}{5}$$

The simplest radical form of $\sqrt{\frac{36}{25}}$ is $\frac{6}{5}$.

Examples

Rewrite each square root in simplest radical form.

1. $\sqrt{\frac{64}{81}} =$

2. $\sqrt{\frac{1}{16}} =$

3. $\frac{\sqrt{45}}{\sqrt{5}} =$

4. $\frac{\sqrt{80}}{\sqrt{20}} =$

Try These

Rewrite each square root in simplest radical form.

1. $\sqrt{\dfrac{1}{100}} =$

2. $\sqrt{\dfrac{49}{625}} =$

3. $\sqrt{\dfrac{25}{9}} =$

4. $\sqrt{\dfrac{36}{25}} =$

5. $\dfrac{\sqrt{11}}{\sqrt{44}} =$

6. $\dfrac{\sqrt{75}}{\sqrt{3}} =$

7. $\dfrac{\sqrt{6}}{\sqrt{54}} =$

8. $\dfrac{\sqrt{8}}{\sqrt{50}} =$

Rationalizing the Denominator

Sometimes the radical in the denominator doesn't fully simplify and you are left with a radical in the denominator… which we can't have!

Rationalizing the Denominator means we are fixing the denominator so that it does not contain an irrational number (in this case a square root).

Examples

1. $\sqrt{\dfrac{25}{3}}$

2. $\dfrac{25}{\sqrt{5}}$

3. $\sqrt{\dfrac{3}{8}}$

4. $\dfrac{10}{\sqrt{2}}$

Try These

Rewrite each square root in simplest radical form.

1. $\sqrt{\frac{2}{5}}$

2. $\sqrt{\frac{8}{5}}$

3. $\frac{2}{\sqrt{10}}$

4. $\frac{3}{\sqrt{18}}$

5. $\frac{\sqrt{3}}{\sqrt{10}}$

6. $\frac{8}{\sqrt{8}}$

Solving Equations Using Square Roots

When you first started to solve equations, you learned to use opposite operations to "undo."

Square Roots $\sqrt{\blacksquare}$ and Squaring $\blacksquare^2$ are opposite operations. They "undo" each other.

Example One

Solve $x^2 - 144 = 0.$
First, we need to get x^2 by itself. Add 144 to both sides.

$$x^2 = 144$$

The equation is asking…what number times itself is equal to 144. We know that $12^2 = 144$ and that $(-12)^2 = 144.$ So, there are **2** answers, one positive and one negative.

Now, let's take the square root of both sides of the equation. Because there are two answers, one positive and one negative, we put this in front of the number.

$$\sqrt{x^2} = \pm\sqrt{144}$$
$$x = \pm 12$$

So $x^2 - 144 = 0$ has two solutions, $x = 12$ or $x = -12$.

Example Two

Solve $\frac{1}{6}x^2 - 4 = 10$.

Try These

1. $4x^2 - 5 = -1$

> **Hint:** Remember to get x by itself **before** you take the square root of both sides of the equation.

2. $x^2 - 200 = 0$

3. $\frac{1}{3}x^2 - 3 = 33$

4. $5x^2 - 32 = 68$

Example Three

Solve $5(x-7)^2 = 125$

Example Four

Solve $\frac{1}{3}(x-4)^2 = 3$

Try These

1. $(x+3)^2 = 9$

> **Hint:** Remember to get $(\quad)^2$ by itself **before** you take the square root of both sides of the equation.

2. $-2(x-3)^2 = -32$

3. $5(x-2)^2 = 20$

4. $\frac{1}{3}(x+5)^2 = 27$

$\pm$