Unit 6 Notes

A **system of linear equations** is a group of two or more linear equations.

We will learn to solve a system of **TWO** linear equations. If we graph two linear equations, we have three possible graphs.

Intersect at a Point	Parallel	Same Line
Solution is the **point of intersection**	There is **no solution**	Solution is **all the points on the line**

The point of intersection makes both equations true and is, therefore, the solution of the system.

Examples

Determine if the following point is a solution of the system.

1. Is $(3, -1)$ a solution of the system $\begin{cases} x - 2y = 5 \\ 2x + y = -5 \end{cases}$?

2. Is $(4, -5)$ a solution of the system $\begin{cases} y = x + 9 \\ 3x + 2y = 2 \end{cases}$?

3. Is $(-2, -4)$ a solution of the system $\begin{cases} y = -\frac{1}{2}x - 5 \\ y = 2x \end{cases}$

Try These

Determine if the following point is a solution of the system.

4. Is $(6, -2)$ a solution of the system $\begin{cases} x + 5y = -4 \\ y = \frac{1}{2}x - 1 \end{cases}$?

5. Is $(-3, 5)$ a solution of the system $\begin{cases} 2x + y = -1 \\ y = \frac{1}{3}x + 6 \end{cases}$?

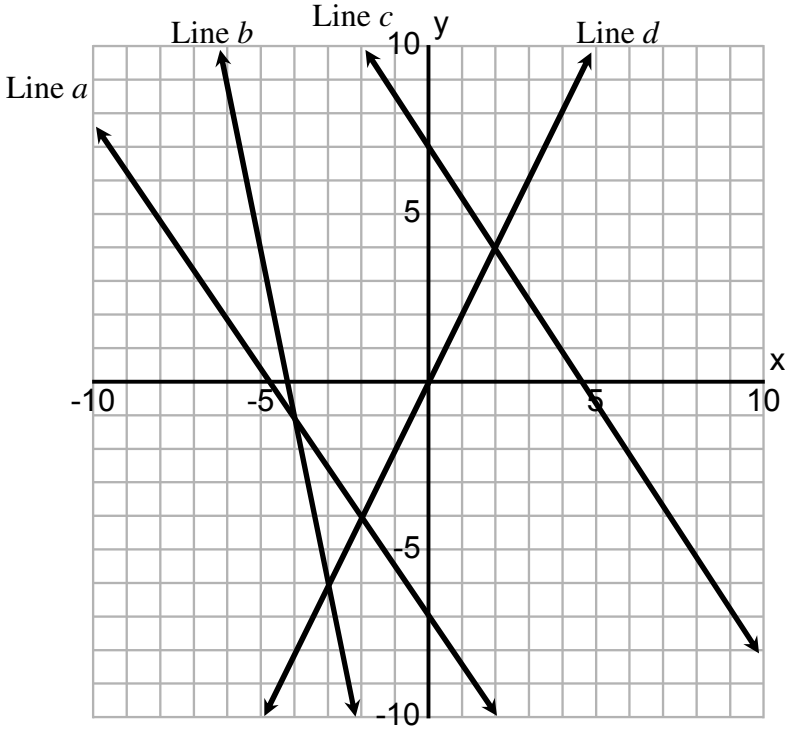
6. Is $(4,\ 8)$ a solution of the system $\begin{cases} 3x - y = 4 \\ 2x - 3y = -16 \end{cases}$

Solving Systems by Graphing

To solve a system graphically, you must graph the equations and see if they INTERSECT. **The** point of intersection **will be the solution of the system.**

Find the solution for each system.

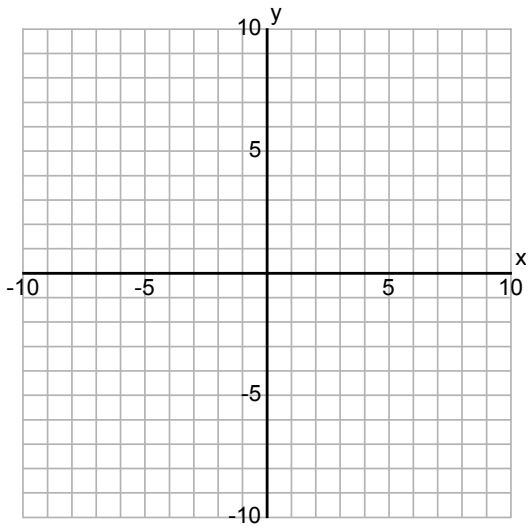
1. Line a & Line b
2. Line a & Line c
3. Line a & Line d
4. Line b & Line d
5. Line c & Line d



Examples

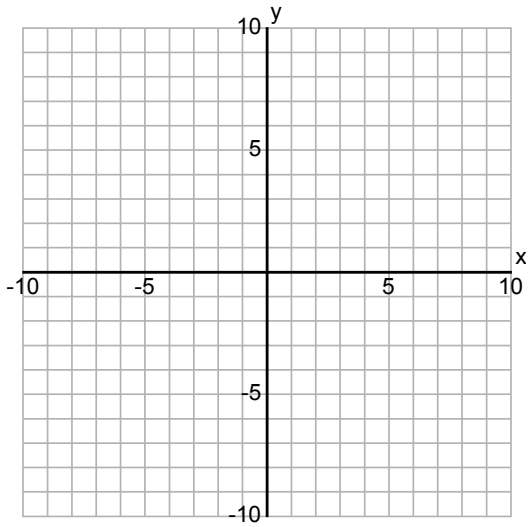
1. Solve the following system of linear equations by graphing.

$$\begin{cases} 2x - y = 8 \\ x + y = 1 \end{cases}$$



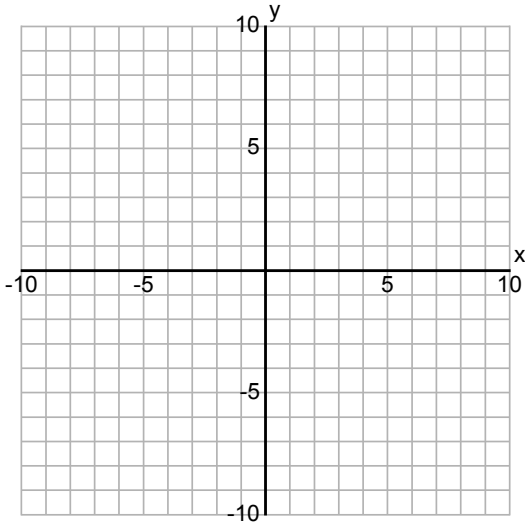
2. Solve the following system of linear equations by graphing.

$$\begin{cases} y = x + 2 \\ y = -3 \end{cases}$$



3. Solve the following system of linear equations by graphing.

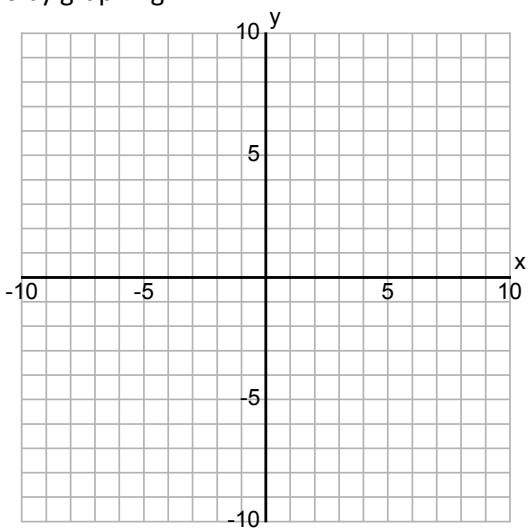
$$\begin{cases} x - 2y = -6 \\ x - 2y = 2 \end{cases}$$



Try These

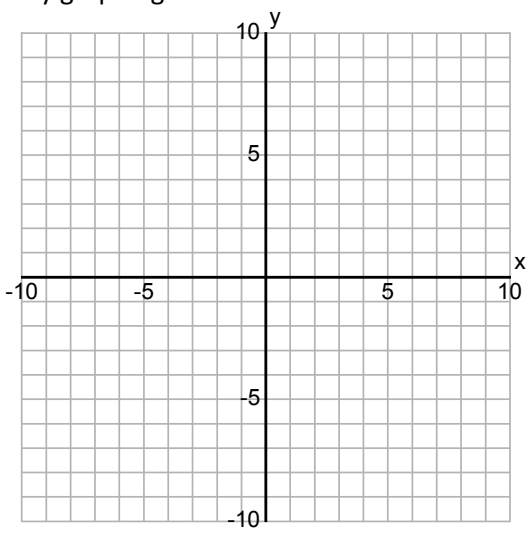
4. Solve the following system of linear equations by graphing.

$$\begin{cases} x + y = -2 \\ 2x - y = 10 \end{cases}$$



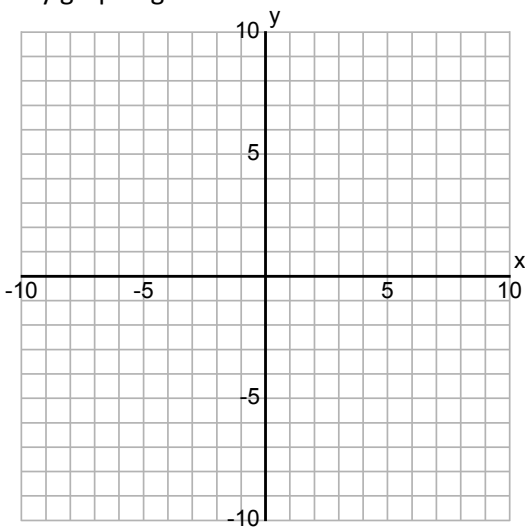
5. Solve the following system of linear equations by graphing.

$$\begin{cases} y = x - 4 \\ x = 5 \end{cases}$$



6. Solve the following system of linear equations by graphing.

$$\begin{cases} 3x - 9y = 3 \\ -x + 3y = -3 \end{cases}$$



Solving Systems by Substitution

Steps for Substitution

1. Solve one of the equations for one of the variables.
2. Substitute the expression from step 1 into the OTHER equation and solve for the other variable.
3. Substitute the value from step 2 into the one of the original equations and find the value of the second variable.
4. Write your solution as an ordered pair. The variables should be written alphabetically.

Examples

1. Solve the following system $\begin{cases} y = -2 \\ 4x - 3y = 8 \end{cases}$

2. Solve the following system $\begin{cases} a + b = 6 \\ a - b = -2 \end{cases}$

Hint: Solve for the variable that has a coefficient of **1** or **–1**.

3. Solve the following system $\begin{cases} 6m + n = -2 \\ m - 3n = -13 \end{cases}$

4. Solve the following system $\begin{cases} 2x - y = 6 \\ 2x + 2y = -9 \end{cases}$

Try These

Solve each system using the substitution method.

1. $\begin{cases} x = 2y + 1 \\ 3x - 4y = -1 \end{cases}$

2. $\begin{cases} 2a - b = 0 \\ a - b = 2 \end{cases}$

3. $\begin{cases} 2m + 3n = 7 \\ m - 2n = -7 \end{cases}$

4. $\begin{cases} y = -2x \\ 3x - y = 15 \end{cases}$

5. $\begin{cases} c + d = -3 \\ 3c + 2d = -10 \end{cases}$

6. $\begin{cases} 6p - q = -8 \\ p - q = 2 \end{cases}$

Solving Systems by Elimination

Steps for Elimination (Linear Combination)

1. Multiply one or both of the equations by a number so that the coefficients of one of the variables are opposites
2. Add the equations from step 1 together. Combining the equations will **eliminate** one of the variables.
3. Solve for the remaining variable.
4. Substitute the value from step 3 into the one of the original equations and find the value of the second variable.
5. Write your solution as an ordered pair. The variables should be written alphabetically.

Examples

1. Solve the following system: $\begin{cases} 4x + 5y = 6 \\ 3x - 5y = 8 \end{cases}$

2. Solve the following system: $\begin{cases} 2m - 3n = 5 \\ 5m - 3n = 11 \end{cases}$

Try Some

3. $\begin{cases} 2a + b = 6 \\ -2a - 3b = 8 \end{cases}$

4. $\begin{cases} 3x + 4y = 7 \\ 2x - 4y = 13 \end{cases}$

5. $\begin{cases} 4g + 5h = 12 \\ 4g - 3h = -4 \end{cases}$

6. $\begin{cases} -p - 3q = -7 \\ 2p - 3q = -4 \end{cases}$

Examples

7. Solve the following system $\begin{cases} 9x + 2y = 0 \\ 3x - 5y = 17 \end{cases}$

Hint: Find the **LCM** of the coefficients for either x or y. The LCM is the smallest number that both coefficients will divide

8. Solve the following system $\begin{cases} 11m + 6n = 1 \\ 3m + 2n = -3 \end{cases}$

9. Solve the following system $\begin{cases} 5x - 2y = -29 \\ 4x + 3y = -14 \end{cases}$

Remember: You are trying to make either the x or y coefficients opposites. Sometimes you have to change **both** equations.

Try These

Solve each system using the elimination method.

10. $\begin{cases} 7x + 3y = -12 \\ -7x + 2y = -8 \end{cases}$

11. $\begin{cases} a + 3b = 8 \\ 4a - 3b = 2 \end{cases}$

12. $\begin{cases} v + 2w = 7 \\ 3v + 5w = 17 \end{cases}$

13. $\begin{cases} 7m - 5n = 12 \\ 2m + n = 1 \end{cases}$

14. $\begin{cases} 2x + 3y = 18 \\ 5x - y = 11 \end{cases}$

15. $\begin{cases} 3e + 2f = 8 \\ 2e - 5f = -20 \end{cases}$

Special Cases when Solving Systems

Consistent System	
Inconsistent System	

Find the solution to each system and determine whether each system is consistent or inconsistent.

1. $\begin{cases} x + y = 7 \\ x + y = -5 \end{cases}$

2. $\begin{cases} 3w + 2y = 5 \\ -3w - 4y = -10 \end{cases}$

3. $\begin{cases} a + b = 5 \\ -2a - 2b = -8 \end{cases}$

4. $\begin{cases} m - n = 2 \\ -m + n = -2 \end{cases}$

5. $\begin{cases} 2x - y = 1 \\ x + y = 5 \end{cases}$

6. $\begin{cases} 4c - 2d = 2 \\ -2c + d = 1 \end{cases}$

Applications of Linear Systems

Examples

1. If you buy six pens and one mechanical pencil, you'll get only $1 change from your $10 bill. But if you buy four pens and two mechanical pencils, you'll get $2 change. How much does each pen and each pencil cost?

2. A car rental agency charges a daily fee plus a cost per mile. If a car driven 40 miles in one day cost $28, and the same car driven 100 miles in one day cost $37, what is the daily fee and the cost per mile?

3. Mr. Schuler decides to have a pizza party for his three algebra classes. He orders 3 pizzas and 3 bottles of soda for $23.34 for his first period class and 4 pizzas and 6 bottles of soda for $32.70 for his fifth period class. If he orders 5 pizzas and 5 bottles of soda for his third period class, how much will it cost?

Try These

4. A caterer's total cost for catering a party includes a fixed cost plus a certain amount for each guest. If it costs $300 to serve 25 guests and $420 to serve 40 guests, find the fixed cost and the cost per guest.

5. Three pizzas and four subs cost $34. Four pizzas and seven subs cost $49.50. How much does a pizza cost?

6. Joe is the place kicker for his football team. Last season he kicked 38 times and never missed. Each field goal he kicked scored 3 points and each point after touchdown scored 1 point for a total of 70 points. How many field goals did Joe kick last season?