Elementary Probability Theory

Probability- A number between 0 and 1 (sometimes a \%) which is used to indicate the likelihood of a given event occurring.

|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  | $\leftarrow$ | --> |  |
| No | Less | More | Will |
| Chance | likely | likely | happen |

Notation: $P(A)$-> the probability of an event " $A$ " occuring

3 methods of assigning probabilities to events:

1) Intuition
2) Relative frequency
3) Equally likely outcomes

Relative Frequency: $P(A)=\#$ of times A occurs total \# of events

Ex 1: A certain genetic defect occurs in 40 out of 100 fruit flies. By method of relative frequency, the probability of a fruit fly being defective is

Ex 2: A field goal kicker has made 7 out of 10 field goals from a distance of $35-45$ yards. Based on this past pattern, we can say that the probability of this kicker making a field goal of this distance is

Note: The assumption is that if events occurred a certain percentage of the time in the past, then they will occur about the same percentage of the times in the future. Law of Large Numbers: As the sample size increases, the relative frequencies of outcomes get closer and closer to the theoretical (or actual) probability value.
** This law is the reason that businesses such as insurance companies, casinos, and lotteries make such huge profits.**

For example: With a slot machine, the winnings of a gamble on a single play or even a few plays are uncertain (small sample). However on millions of plays the actual probability favors the casino. The casino is guaranteed to profit!

Equally Likely Outcomes: P(event) = \# of outcomes favorable to event
total number of outcomes

Ex 3: A bag contains 10 red and 5 white marbles. A marble is selected from a bag at random. What is

> P(red)

## P(white)

Ex 4: A multiple choice question has 4 possible responses. What is the probability of "guessing" the correct answer?

## Acey/ Ducey Game

In your pairs you will start each game by each of you choosing one card at random from the deck (write these in the table). Find the probability of the next card drawn being between the two cards you chose (ace counts as an 11). Write down the probability in the table knowing that there are 50 cards left in the deck with four of each type of card. Draw a card and write down if you won or lost (you win if the card is between the two you have). Be sure to shuffle the cards back into the deck before starting the next draw.

| Card \#1 | Card \#2 | Probability of <br> winning | Card \#3 | Win or Lose? |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |


|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

Highlight all the winning games where the probability was $24 \%(12 / 50)$ or higher. How many winning games do you have that are not highlighted?

Put a star next to the losing games where the probability was $48 \%(24 / 50)$ or higher.

The games you highlighted won with a $24 \%$ chance or higher of winning. The winning games that are not highlighted had a very low chance of winning yet you beat the probability. The games you starred had a high chance of winning (48\% or higher) and yet you lost.

1) What does this activity say about the validity of probability to predict your chances of winning or losing?
2) Look through your table, do you have any games with the probability of winning being $80 \%$ (40/50) or better?

If so, did you win those games?
3) If you drew a 3 on the first card, what would you have to draw on the second card to have a probability of 0 ?
4) If you drew a 2 on the first draw and an ace on the second draw would you have a probability of $100 \%$ ?

If not, what would the probability be?
5) Compare your game results to another group's. Which group won more often?

Did this group have higher chances of winning or did they just beat the odds more often?

Practice for Elementary Probability.
A. Calculate the probability of the given event.

1. An NBA player has made 125 out of 150 free throws this season so far. What is the probability that he will make his next free throw?
2. A card is selected at random from a standard deck of 52 playing cards. What is the probability that
a. the card is an ace?
b. the card is red (heart or diamond)?
c. the card is a diamond?
d. the card is the ace of spades?
e. the card is a face card (jack, king, or queen)?
f. the card has an even number?
g. the card is a black king?
3. A baseball player has 20 homeruns in 500 at bats. What is the probability that he will hit a homerun in his next at bat?
4. A doctor correctly diagnosed 48 out of 50 patients with a certain virus.
a. What is the probability that a patient with this virus will be correctly diagnosed?
b. What is the probability that a patient with this virus will be incorrectly diagnosed?
5. A bag contains 20 nickels, 18 dimes and 12 quarters. A coin is selected at random from the bag. What is the probability that the coin is
a. a nickel?
b. a dime?
c. a quarter?
d. a nickel or a dime?
e. a nickel or dime or quarter?
f. a dollar
6) Use the following data to answer the questions below the table:

| Number of Fires of Various Types <br> Among 100 Typical Home Fires |  |
| :--- | :---: |
| Cause of the fire | Number reported |
| Heating System | 22 |
| Cooking | 15 |
| Electrical System | 8 |
| Smoking | 7 |
| Appliances | 7 |
| Other | 41 |
| Total | 100 |

a) What is the probability of a fire starting due to a cooking fire?
b) What is the probability of a fire starting due to electrical wiring?
c) What is the probability of a fire starting due to something other than smoking?
d) What is the probability of a fire starting due to the heating system or appliances?
e) What is the probability of a fire starting due to someone cooking or smoking?
f) Can we estimate the probability that the fire was caused by lightning?
7) Use the month of January (31 days) to answer the following questions.
a) If you randomly select a day in January, what is the probability of you choosing January $17^{\text {th }}$ ?
b) If you randomly select a day in January, what is the probability of you choosing an odd numbered day?
c) If you randomly select a day in January, what is the probability of you choosing a day numbered with a single digit?
8) Suppose the ten digits from 0 to 9 are written on ten poker chips, one number per chip. The chips are then placed in a box and mixed. You reach into the box and pull out one chip. What is the probability that the number on the chip is:
a) a 1 ?
b) an even number ( 0 is considered even)?
c) a number greater than 7 ?
d) a number divisible by 3 ( 0 is divisible by 3 )?

